Constraint Handling Rules

A Tutorial for (Prolog) Programmers

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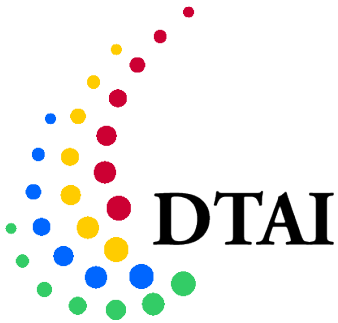

About This Tutorial

Expected background:

- know & like programming

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- but have an itch that Prolog doesn't scratch

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Expected background:

- know & like programming
- know Prolog, ...
- but have an itch that Prolog doesn't scratch
- ...or you will have at the end of this tutorial

About This Tutorial

Expected mindset:

- forget about Prolog for now

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You've been warned! Also:

- do not expect anything from the word "constraint"

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- forget about Prolog for now
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- really, they won't be the same!!!

You've been warned! Also:

- do not expect anything from the word "constraint"
- CHR constraints aren't

About this tutorial

What you'll learn:

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1. how CHR works (operationally)

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2. how to program in CHR

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What you'll learn:

1. how CHR works (operationally)
2. how to program in CHR
3. just maybe, what CHR is good for

Art 101: Mixing Paint

As novice, you have to start at the bottom of the ladder:

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- Learning how to obtain different colors

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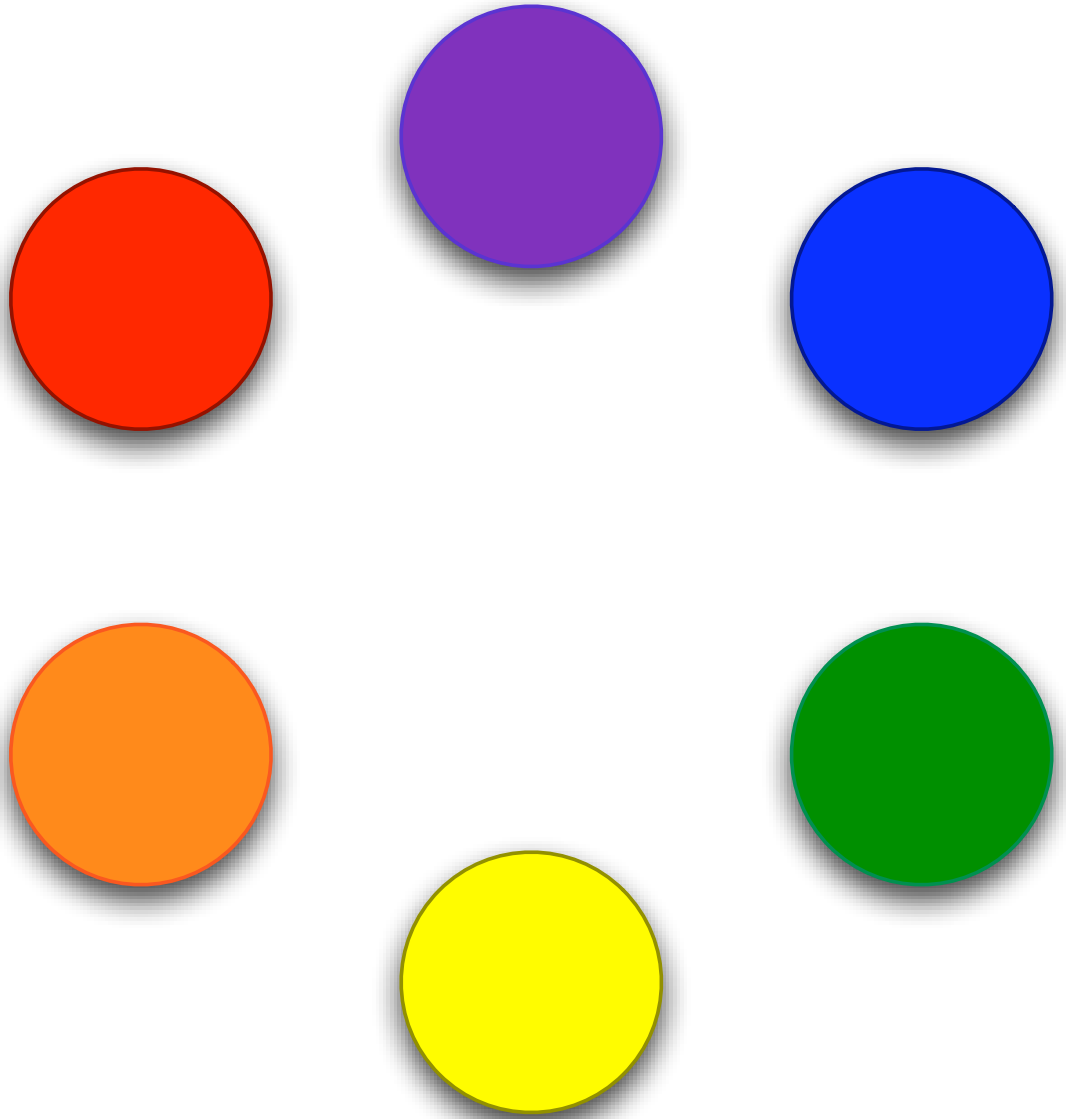
- by mixing paint

Art 101: Mixing Paint

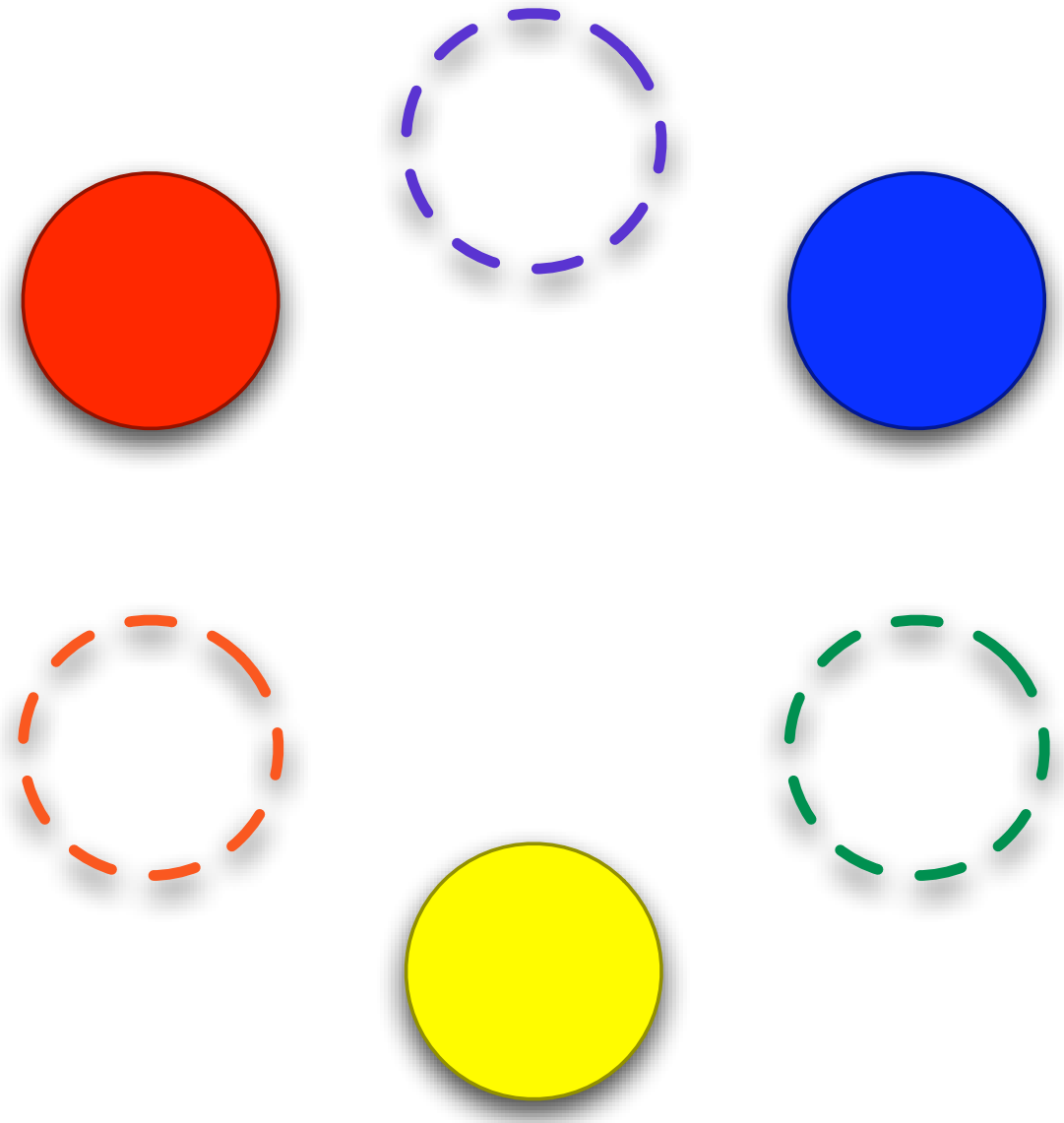
As novice, you have to start at the bottom of the ladder:

- Learning how to obtain different colors
- by mixing paint
- Made easy by the Paint Mixing Rules (PMR) language

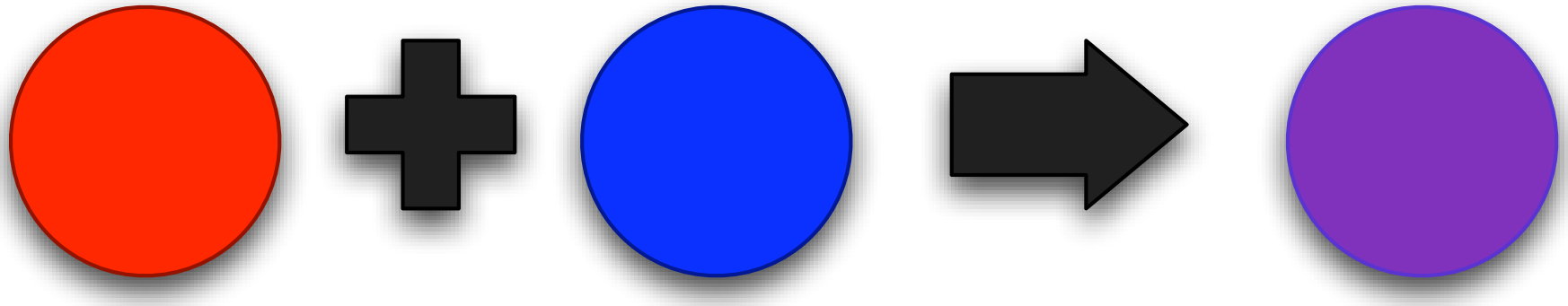
The Paint



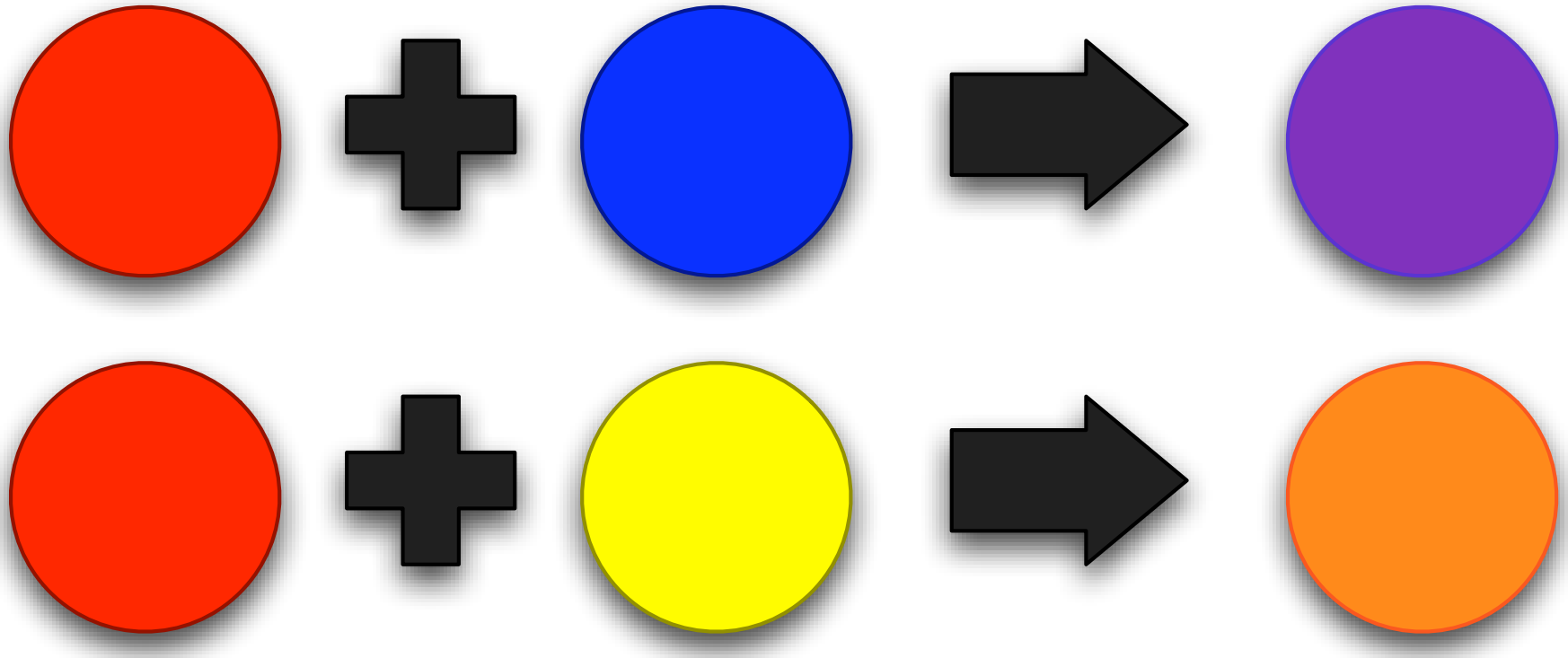
Fewer Paint



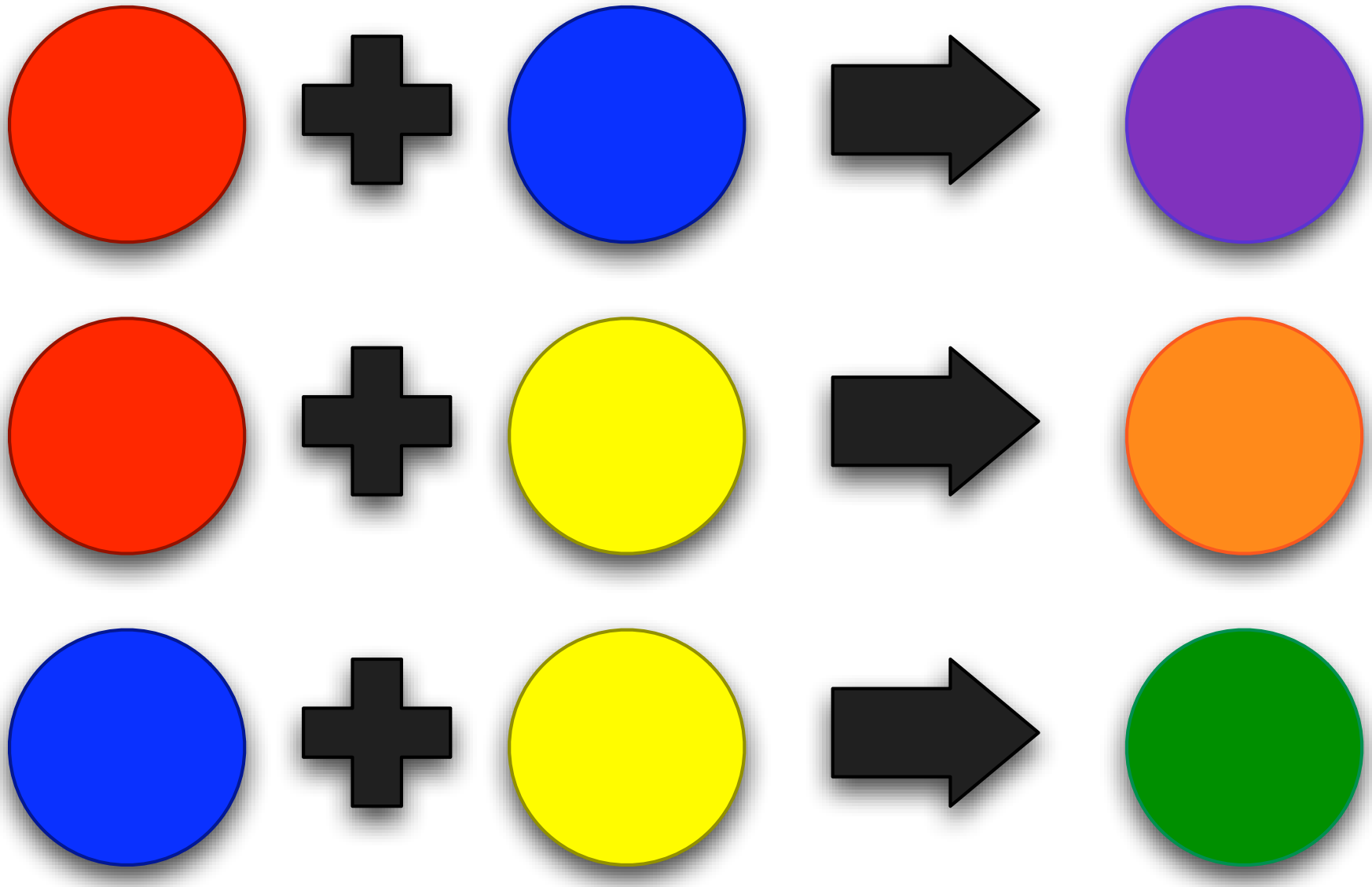
The Rules



The Rules



The Rules



PMR as CHR

Under the hood, PMR is of course CHR:

PMR	**CHR**
paints	constraints
rules	simplification rules (textual)
mixing bucket	constraint store

Paint as Constraints

Declaring our paint colors, in textual form:

```
:- chr_constraint red.                    %
```

Paint as Constraints

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```
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:- chr_constraint blue.                   %
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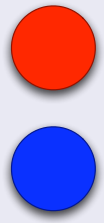
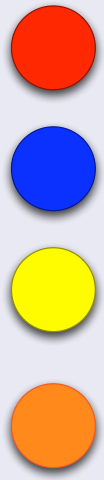


Paint as Constraints

Declaring our paint colors, in textual form:

```
:- chr_constraint red.                    %
:- chr_constraint blue.                   %
:- chr_constraint yellow.                 %
:- chr_constraint orange.                 %

...
```



Paint as Constraints?

Are paint colors really “constraints”?

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Yes: in the CHR sense

- 1st-class entities in CHR
- rewritten by rules

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No: not in the Constraint Programming sense

Yes: in the CHR sense

- 1st-class entities in CHR
- rewritten by rules

(Are Prolog predicates really "predicates"?)

The Paint Rules in CHR

Simplification Rule: *head* <=> *body* .

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red, blue    <=> purple.
```

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The Paint Rules in CHR

Simplification Rule: *head* <=> *body* .

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```
red, blue     <=> purple.
```

% 

```
red, yellow   <=> orange.
```

% 

```
blue, yellow <=> green.
```

Time For...

...a cocktail break!

Cocktail Break

You can have as many heads as you like:

```
vodka, martini <=> vodka_martini.
```

Cocktail Break

You can have as many heads as you like:

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vodka, martini <=> vodka_martini.

tequila, cointreau, lime <=> margarita.
```

Cocktail Break

You can have as many heads as you like:

```
vodka, martini <=> vodka_martini.

tequila, cointreau, lime <=> margarita.

gin, cherry_brandy, cointreau, grenadine,
pineapple, lemon, angostura_bitters
  <=> singapore_sling.
```

but at least one!

Cocktail Break

Warning: too many heads may cause a headache!

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```
vodka_martini, margarita, singapore_sling
        <=> hangover, blackout.
```

Cocktail Break

Warning: too many heads may cause a headache!

```
vodka_martini, margarita, singapore_sling
        <=> hangover, blackout.
```

You can have many constraints in the body too.

Running the Rules

Paint-Mixing Engine:

- Prolog (SWI-Prolog, . . .)
- Leuven CHR system

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Everything in a file:

paint.pl

```
:- use_module(library(chr)).

:- chr_constraint red.
...

red, blue <=> purple.
...
```

Running the Rules

Paint-Mixing Engine:

- Prolog (SWI-Prolog, . . .)
- Leuven CHR system

Everything in a file:

paint.pl

```
:- use_module(library(chr)).

:- chr_constraint red.
...

red, blue <=> purple.
...
```

```
?- [paint].
```

Running the Rules

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

```
?- red, blue.
```

- queries
- answers

Running the Rules

```
red, blue    <=> purple.
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```

- queries
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```
?- red, blue.
purple

?- red, yellow.
```

Running the Rules

```
red, blue   <=> purple.
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```

- queries
- answers

```
?- red, blue.
purple

?- red, yellow.
orange

?- blue, yellow.
```

Running the Rules

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red, blue   <=> purple.
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```

- queries
- answers

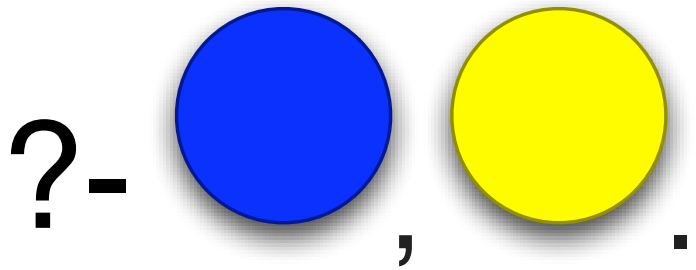
```
?- red, blue.
purple

?- red, yellow.
orange

?- blue, yellow.
green
```

In Slow Motion

```
red, blue   <=> purple.
red, yellow <=> orange.
blue, yellow <=> green.
```



- query: left-to-right

In Slow Motion

```
red, blue    <=> purple.
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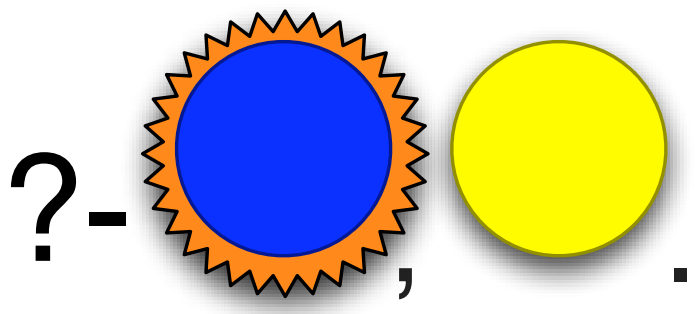
```



- query: left-to-right
- active constraint: `blue`

In Slow Motion

```
red, blue    <=> purple.
red, yellow  <=> orange.
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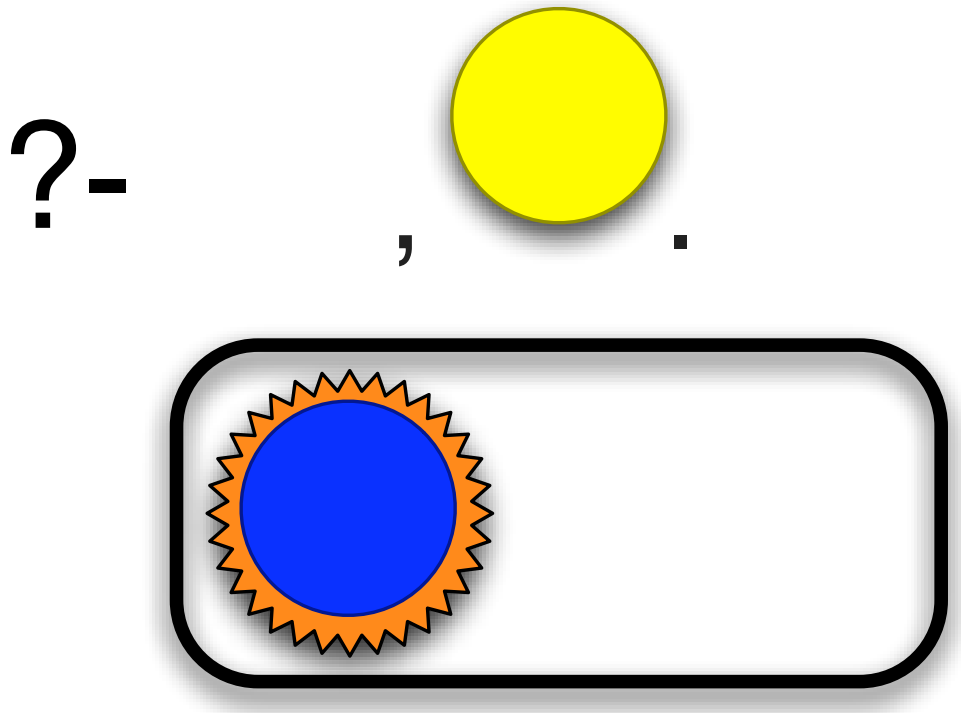
```



- query: left-to-right
- active constraint: `blue`
- put in constraint store

In Slow Motion

```
red, blue   <=> purple.
red, yellow <=> orange.
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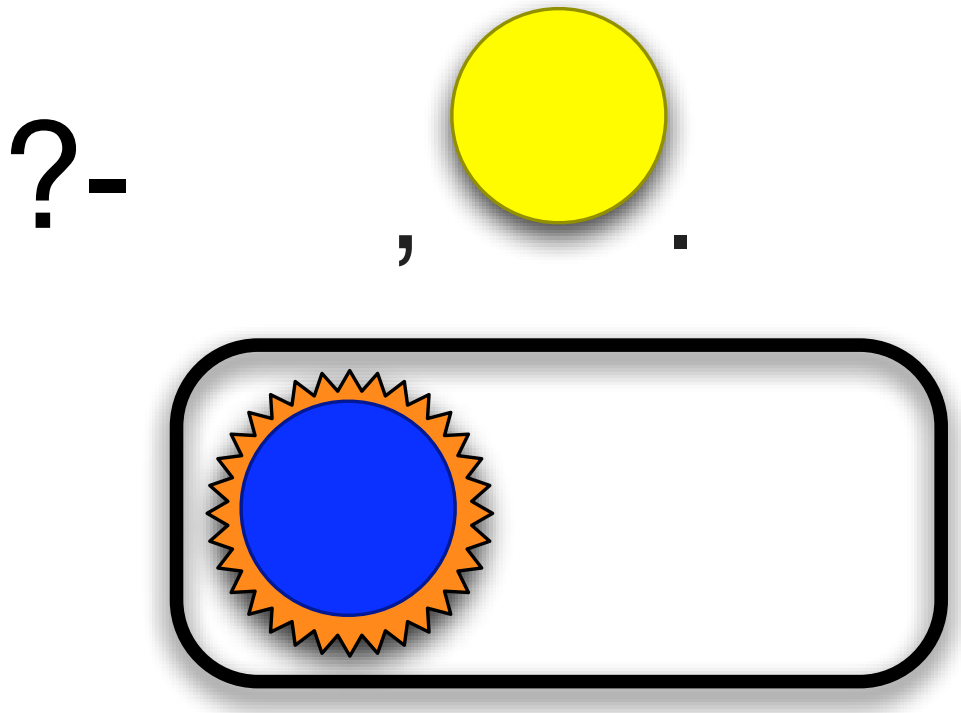
```



- active constraint: `blue`
- fire first rule?
- no partner `red` in constraint store

In Slow Motion

```
red, blue    <=> purple.
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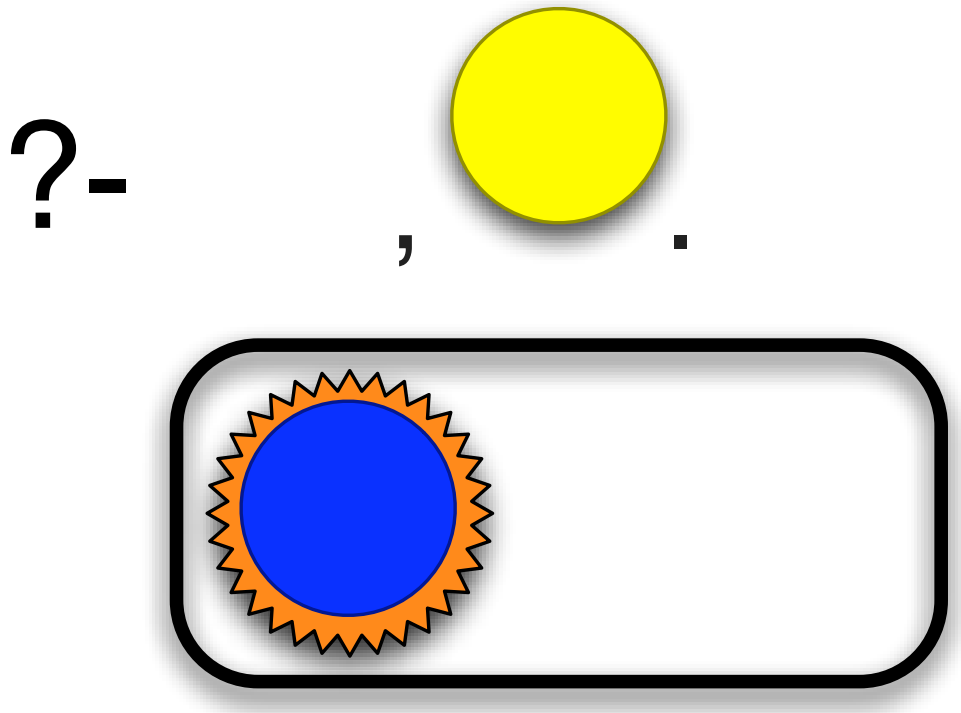
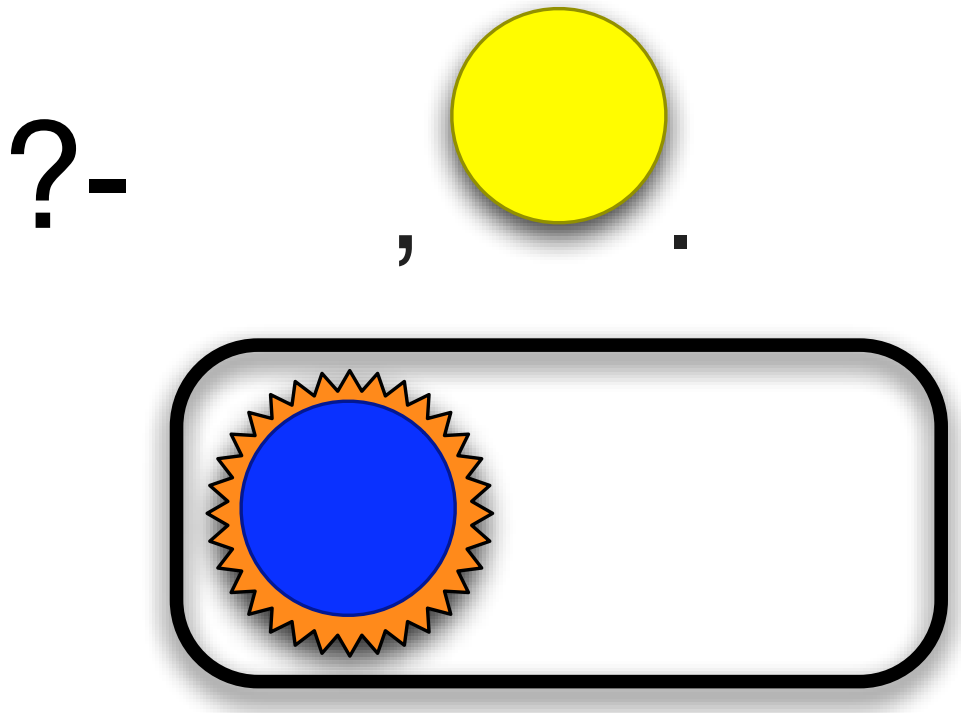


- active constraint: `blue`
- fire second rule?
- no `blue` in head

In Slow Motion

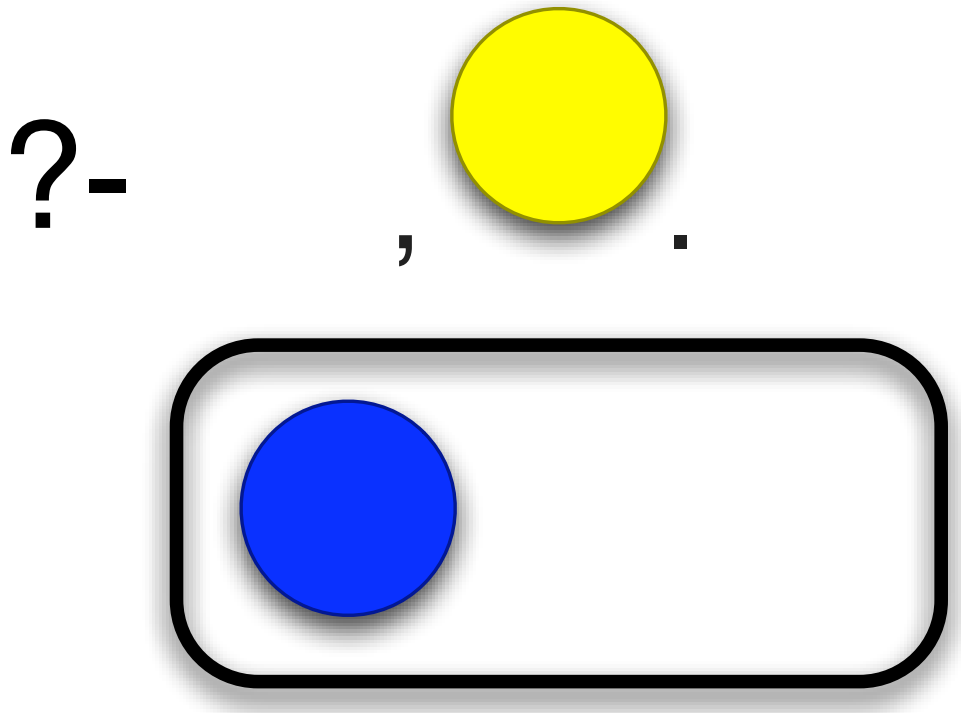
```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```



- active constraint: `blue`
- fire third rule?
- no partner `yellow` in constraint store

In Slow Motion

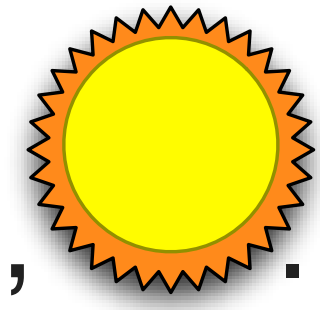
```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

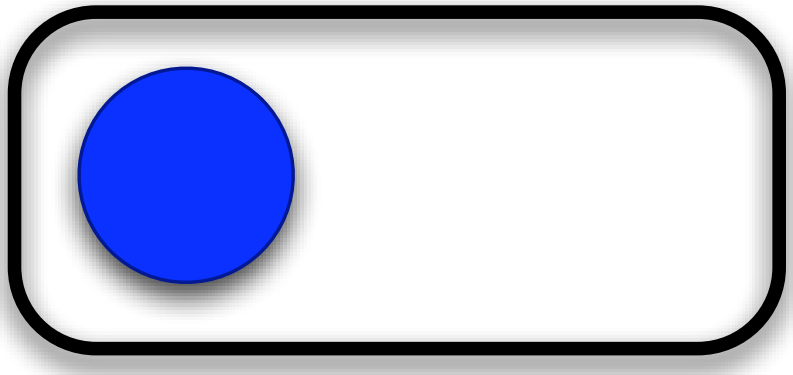


- deactivate constraint `blue`

In Slow Motion

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

?-  .

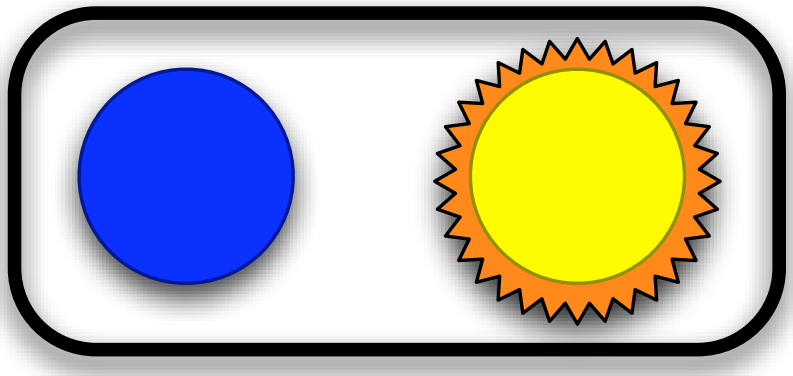


- active constraint: `yellow`

In Slow Motion

```
red, blue   <=> purple.
red, yellow <=> orange.
blue, yellow <=> green.
```

?- , .

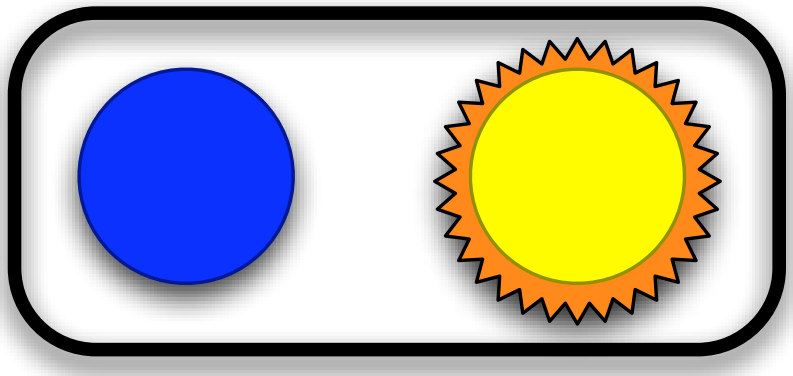


- active constraint: `yellow`
- put in constraint store

In Slow Motion

```
red, blue   <=> purple.
red, yellow <=> orange.
blue, yellow <=> green.
```

?- , .

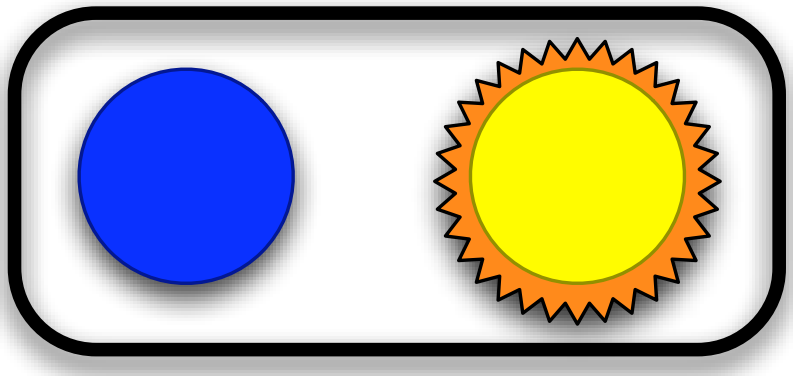


- active constraint: `yellow`
- fire first rule?
- no `yellow` in head

In Slow Motion

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

?- , .



- active constraint: `yellow`
- fire second rule?
- no partner `red` in constraint store

In Slow Motion

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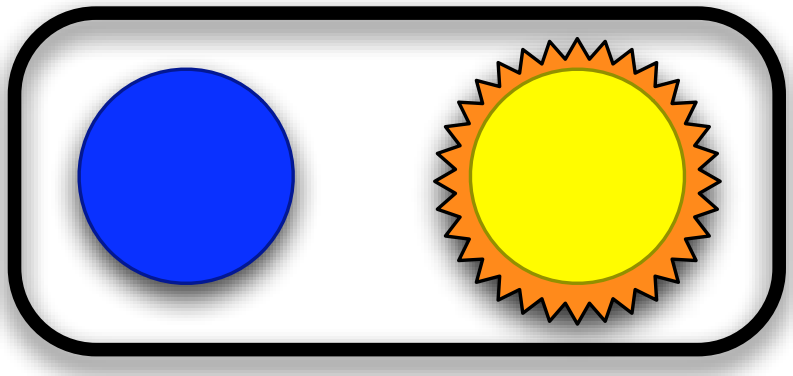
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

?- , .



- active constraint: `yellow`
- fire third rule?

In Slow Motion

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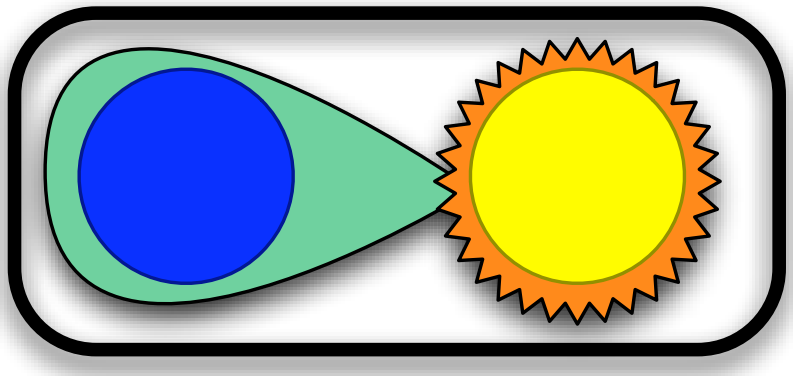
```

?- , .



- active constraint: `yellow`
- fire third rule!
- with partner `blue` in constraint store

In Slow Motion

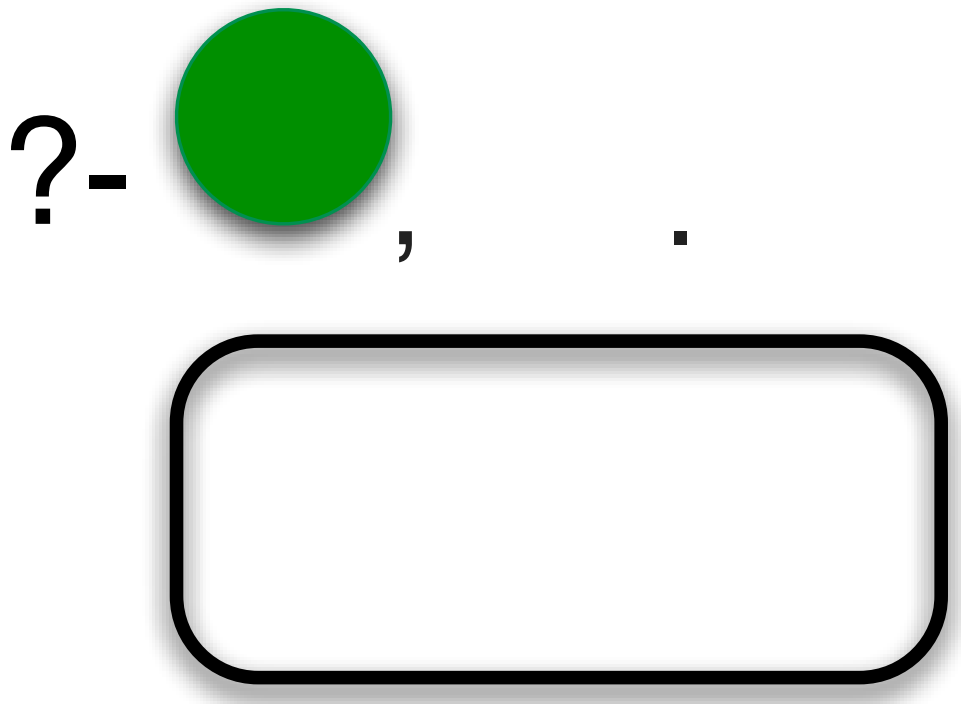
```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

?- , .

- fire third rule!
- delete the matched head constraints

In Slow Motion

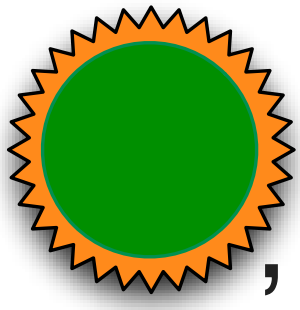
```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```



- fire third rule!
- add the body to (front of) query

In Slow Motion

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

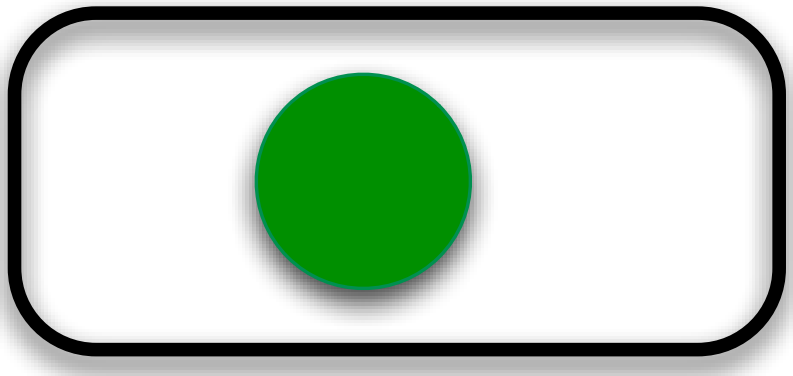
?-  , .

- active constraint: `green`

In Slow Motion

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

?- , .



to make a long story short:

- ...
- final constraint store

What Happens?

```
red, blue   <=> purple.
red, yellow <=> orange.
blue, yellow <=> green.
```

```
?- red, yellow.
orange
```

What Happens?

```
red, blue   <=> purple.
red, yellow <=> orange.
blue, yellow <=> green.
```

```
?- red, yellow.
orange

?- yellow, red.
orange
```

What Happens?

```
red, blue   <=> purple.
red, yellow <=> orange.
blue, yellow <=> green.
```

```
?- red.
```

What Happens?

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

```
?- red.
red
```

What Happens?

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

```
?- red, yellow, blue.
```

What Happens?

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

```
?- red, yellow, blue.
orange
blue
```

What Happens?

```
red, blue   <=> purple.
red, yellow <=> orange.
blue, yellow <=> green.
```

```
?- red, blue, yellow.
```

What Happens?

```
red, blue    <=> purple.
red, yellow  <=> orange.
blue, yellow <=> green.
```

```
?- red, blue, yellow.
purple
yellow
```

Summary

Simplification Rule:

- *head* <=> *body* .
- replace head with body

Summary

Simplification Rule:

- *head* <=> *body* .
- replace head with body

Queries:

- processed left-to-right
- incrementally
- active constraint looks for partners

Brown

Brown stays brown.

```
brown, orange <=> brown.
brown, green  <=> brown.
brown, purple <=> brown.
...
```

Short Hand

Simpagation rule: $head_k \setminus head_r$ <=> $body$.

```
brown \ orange <=> true.
brown \ green  <=> true.
brown \ purple <=> true.
...
```

Short Hand

Simpagation rule: $head_k \setminus head_r$ <=> $body$.

```
brown \ orange <=> true.
brown \ green  <=> true.
brown \ purple <=> true.
...
```

- $head_k$: kept head (1 or more)

Short Hand

Simpagation rule: $head_k \setminus head_r$ <=> $body$.

```
brown \ orange <=> true.
brown \ green  <=> true.
brown \ purple <=> true.
...
```

- $head_k$: kept head (1 or more)
- $head_r$: removed head (1 or more)

Short Hand

Simpagation rule: $head_k \setminus head_r$ <=> $body$.

```
brown \ orange <=> true.
brown \ green  <=> true.
brown \ purple <=> true.
...
```

- $head_k$: kept head (1 or more)
- $head_r$: removed head (1 or more)
- `true` : Prolog's no-op

Lead to Gold

```
philosophers_stone \ lead
        <=> gold.
```

```
?- philosophers_stone, lead.
```

Lead to Gold

```
philosophers_stone \ lead
        <=> gold.
```

```
?- philosophers_stone, lead.
philosophers_stone
gold
```

Lead to Gold

```
philosophers_stone \ lead
        <=> gold.
```

```
?- philosophers_stone, lead.
philosophers_stone
gold

?- philosophers_stone, lead, lead.
```

Lead to Gold

```
philosophers_stone \ lead
        <=> gold.
```

```
?- philosophers_stone, lead.
philosophers_stone
gold

?- philosophers_stone, lead, lead.
philosophers_stone
gold
gold
```

All Combinations

```
philosophers_stone \ lead
        <=> gold.
```

```
?- lead, lead.
```

All Combinations

```
philosophers_stone \ lead
        <=> gold.
```

```
?- lead, lead.
lead
lead
```

All Combinations

```
philosophers_stone \ lead
        <=> gold.
```

```
?- lead, lead.
lead
lead

?- lead, lead, philosophers_stone.
```

All Combinations

```
philosophers_stone \ lead
        <=> gold.
```

```
?- lead, lead.
lead
lead

?- lead, lead, philosophers_stone.
philosophers_stone
gold
gold
```

All Combinations

```
philosophers_stone \ lead
        <=> gold.
```

```
?- lead, lead.
lead
lead

?- lead, lead, philosophers_stone.
philosophers_stone
gold
gold
```

Rule fires with all combinations!

Summary

Simplification Rule:

- *head* <=> *body* .
- replace head with body

Summary

Simplification Rule:

- *head* <=> *body* .
- replace head with body

Simpagation Rule:

- $head_k \setminus head_r$ <=> *body* .
- replace $head_r$ with *body*
- in the presence of $head_k$
- fires all possible combinations

Piggy Bank Merger

```
:- chr_constraint piggy/1.

piggy(I), piggy(J) <=> K is I + J, piggy(K).
```

- constraints can have arguments

Piggy Bank Merger

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- arity must be declared

Piggy Bank Merger

```
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piggy(I), piggy(J) <=> K is I + J, piggy(K).
```

- constraints can have arguments
- arity must be declared
- Prolog terms as arguments

Piggy Bank Merger

```
:- chr_constraint piggy/1.

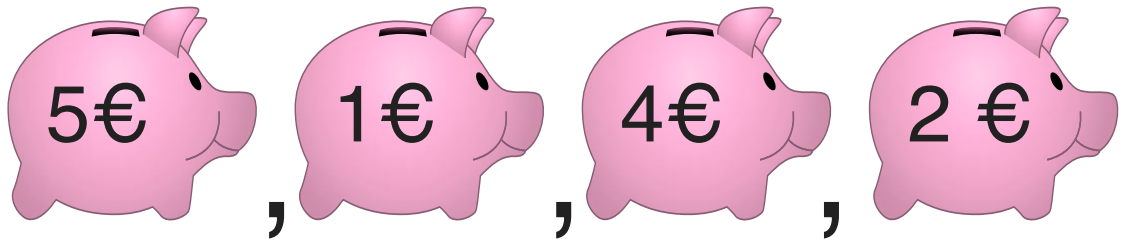
piggy(I), piggy(J) <=> K is I + J, piggy(K).
```

?- piggy(5),piggy(1),piggy(4),piggy(2).

Piggy Bank Merger

```
:- chr_constraint piggy/1.

piggy(I), piggy(J) <=> K is I + J, piggy(K).
```

?- 

.

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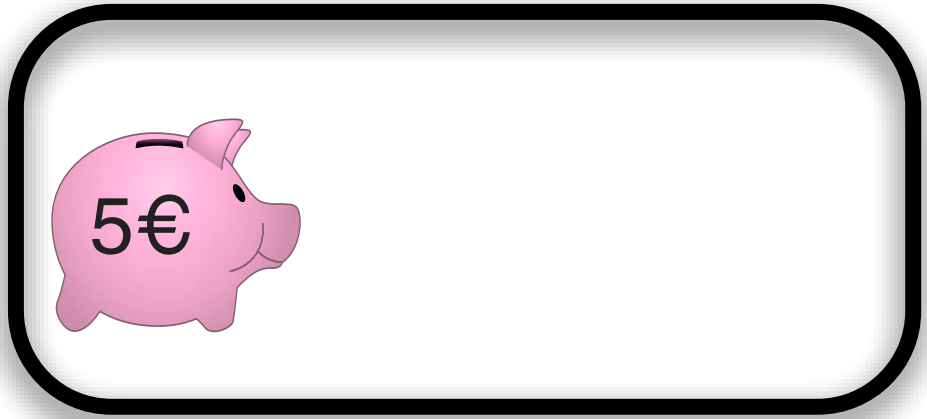
?- 

.



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, 

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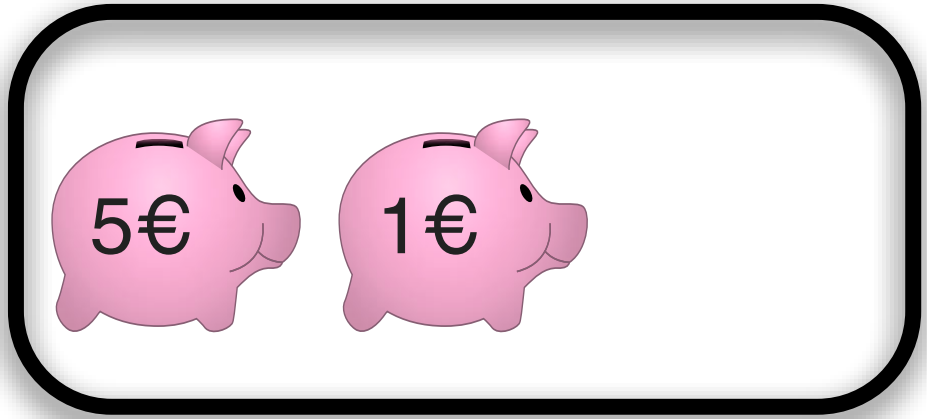

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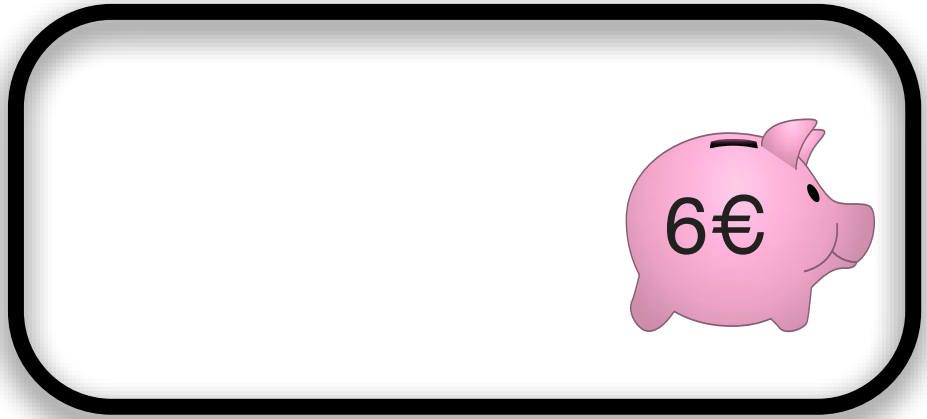

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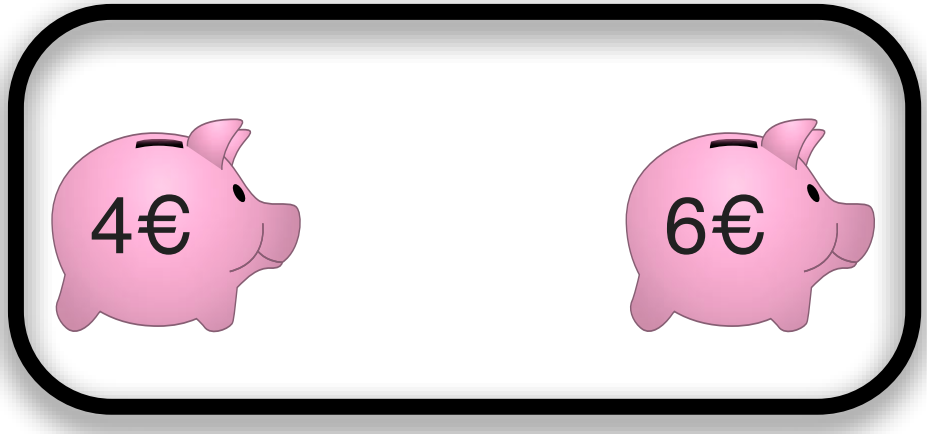

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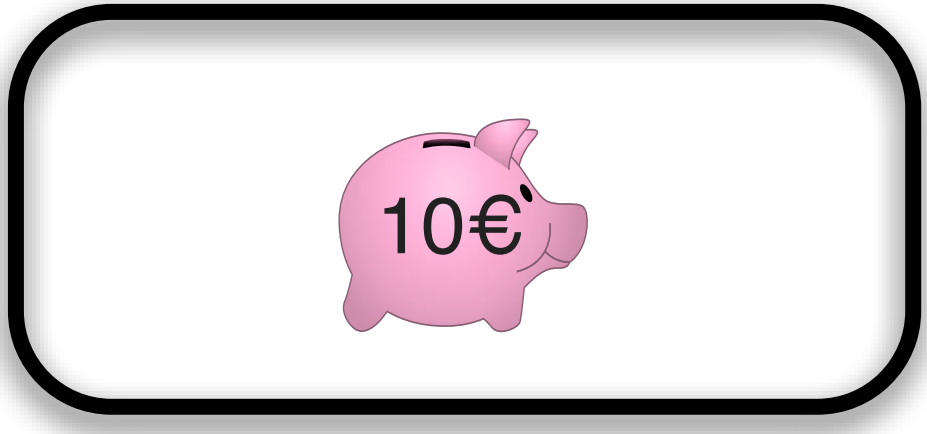

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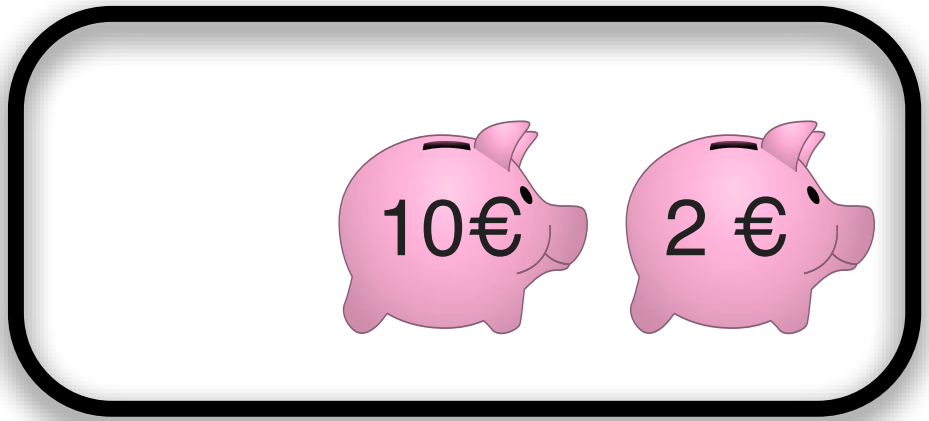

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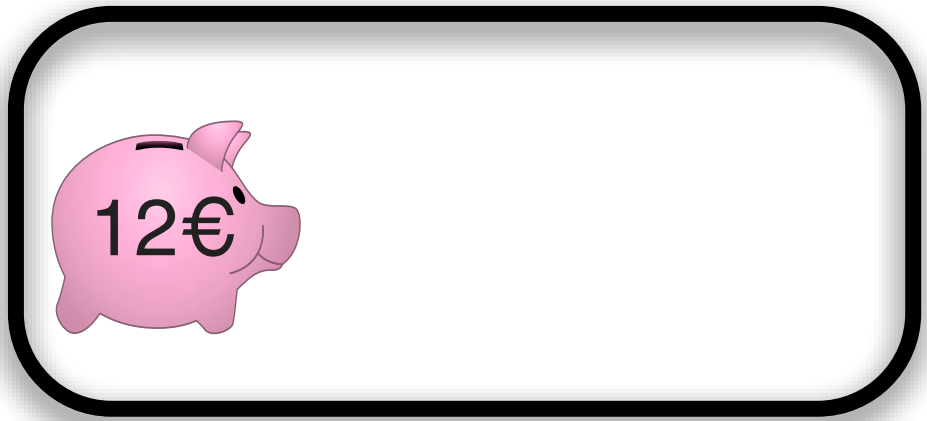

CHR($\mathcal{X}$)

CHR is an embedded language

- embedded in host language $\mathcal{X}$

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- embedded in host language $\mathcal{X}$
- here $\mathcal{X}$ = Prolog

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2-way communication:

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2-way communication:

- Prolog calls CHR constraints
  e.g. all along from toplevel

CHR($\mathcal{X}$)

CHR is an embedded language

- embedded in host language $\mathcal{X}$
- here $\mathcal{X}$ = Prolog

2-way communication:

- Prolog calls CHR constraints
  e.g. all along from toplevel
- CHR calls Prolog
  e.g. `K is I + J`

Not Just Numbers

```
:- chr_constraint value/1.

value(I), value(J) <=> append(I,J,K), value(K).
```

Not Just Numbers

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value(I), value(J) <=> append(I,J,K), value(K).
```

```
?- value([a]), value([b]).
```

Not Just Numbers

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value(I), value(J) <=> append(I,J,K), value(K).
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?- value([a]), value([b]).
value([b,a]).
```

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value(I), value(J) <=> append(I,J,K), value(K).
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?- value([a]), value([b]).
value([b,a]).

?- value([f(a),g(b)]), value([h(i,j)]).
```

Not Just Numbers

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?- value([a]), value([b]).
value([b,a]).

?- value([f(a),g(b)]), value([h(i,j)]).
value([h(i,j),f(a),g(b)]).
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Not Just Numbers

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value(I), value(J) <=> append(I,J,K), value(K).
```

```
?- value([a]), value([b]).
value([b,a]).

?- value([f(a),g(b)]), value([h(i,j)]).
value([h(i,j),f(a),g(b)]).
```

Any Prolog terms!

Summary

CHR constraints

- zero or more arguments
- Prolog terms

Summary

CHR constraints

- zero or more arguments
- Prolog terms

Bodies

- CHR constraints and
- Prolog predicate calls

Count Down

Problem: write a program to enumerate values:

```
?- generate(5).
```

Count Down

Problem: write a program to enumerate values:

```
?- generate(5).
value(5)
value(4)
value(3)
value(2)
value(1)
```

Count Down

Solution:

```
:- chr_constraint generate/1.
```

Count Down

Solution:

```
:- chr_constraint generate/1.
:- chr_constraint value/1.
```

Count Down

Solution:

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=>
  value(N), M is N - 1, generate(M).
```

Running the Program

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=> value(N), M is N - 1, generate(M).
```

```
?- generate(5).
```

Running the Program

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=> value(N), M is N - 1, generate(M).
```

```
?- generate(5).
    % waiting
```

Running the Program

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=> value(N), M is N - 1, generate(M).
```

```
?- generate(5).
    % waiting
    % still waiting
```

Running the Program

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=> value(N), M is N - 1, generate(M).
```

```
?- generate(5).
    % waiting
    % still waiting
ERROR: Out of local stack
```

Running the Program

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=> value(N), M is N - 1, generate(M).
```

```
?- generate(5).
    % waiting
    % still waiting
ERROR: Out of local stack
```

Recursion without a base case!

Guarded Rules

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=> N == 0 | true.
generate(N) <=> N > 0  | value(N),
                         M is N - 1, generate(M).
```

Guarded Rules

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(N) <=> N == 0 | true.
generate(N) <=> N > 0  | value(N),
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Guarded rule:

- apply rule **if** guard succeeds

Guarded Rules

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- guard is optional

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generate(N) <=> N > 0  | value(N),
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```

Guarded rule:

- apply rule **if** guard succeeds
- guard is optional
- guard may contain any Prolog, but may not bind any variables from the head (pure check)

Even Shorter

```
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

Even Shorter

```
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

```
?- generate(5).
```

Even Shorter

```
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

```
?- generate(5).
value(5)
value(4)
value(3)
value(2)
value(1)
generate(0)
```

Summary

Simplification Rule:

- *head* <=> *guard* | *body* .
- replace head with body
- if *guard* succeeds

Summary

Simplification Rule:

- *head* <=> *guard* | *body* .
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Simpagation Rule:

- $head_k \setminus head_r$ <=> *guard* | *body* .
- replace $head_r$ with *body*
- in the presence of $head_k$
- if *guard* succeeds

CHR Style: Matching

```
:- chr_constraint generate/1.
:- chr_constraint value/1.

generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

Matching:

- "inline" notation for guard
- same meaning as explicit guard

Meaning of Matching

Maching	**Guard**
`c(X) <=> true.`	`c(X) <=> true	true.`

Meaning of Matching

Maching	**Guard**
`c(X) <=> true.`	`c(X) <=> true | true.`
`c(a) <=> true.`	`c(X) <=> X == a | true.`

Meaning of Matching

Maching	Guard
`c(X) <=> true.`	`c(X) <=> true | true.`
`c(a) <=> true.`	`c(X) <=> X == a | true.`
`c(f(A)) <=> true.`	`c(X) <=> nonvar(X), X = f(A) | true.`

Meaning of Matching

Maching	Guard
`c(X) <=> true.`	`c(X) <=> true | true.`
`c(a) <=> true.`	`c(X) <=> X == a | true.`
`c(f(A)) <=> true.`	`c(X) <=> nonvar(X), X = f(A) | true.`
`c(X,X) <=> true.`	`c(X,Y) <=> X == Y | true.`

Meaning of Matching

Maching	**Guard**
`c(X) <=> true.`	`c(X) <=> true | true.`
`c(a) <=> true.`	`c(X) <=> X == a | true.`
`c(f(A)) <=> true.`	`c(X) <=> nonvar(X), X = f(A) | true.`
`c(X,X) <=> true.`	`c(X,Y) <=> X == Y | true.`
`c(X), d(X) <=> true.`	`c(X), d(Y) <=> X == Y | true.`

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
No
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
No
```

```
?- c(world).
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
No
```

```
?- c(world).
Hello, World!
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
No
```

```
?- c(world).
Hello, World!

?- c(Free).
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
No
```

```
?- c(world).
Hello, World!

?- c(Free).
c(Free)
```

Matching: CHR vs. Prolog

```
p(world) :- writeln('Hello, World!').

:- chr_constraint c/1.
c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
No
```

```
?- c(world).
Hello, World!

?- c(Free).
c(Free)



?- c(hello).
```

Matching: CHR vs. Prolog

```
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c(world) <=> writeln('Hello, World!').
```

```
?- p(world).
Hello, World!

?- p(Free).
Hello, World!
Free = world

?- p(hello).
No
```

```
?- c(world).
Hello, World!

?- c(Free).
c(Free)


?- c(hello).
c(hello)
```

Summary

Matching

- short-hand for equality-based guards
- one-way unification
  - only instantiates head
  - does not instantiate constraints

Combining Rules

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).

value(I), value(J) <=> K is I + J, value(K).
```

What's the outcome?

```
?- generate(4).
```

Combining Rules

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).

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```

What's the outcome?

```
?- generate(4).
value(10)
```

Combining Rules

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                        M is N - 1, generate(M).

value(I), value(J) <=> K is I + J, value(K).
```

What's the outcome?

```
?- generate(4).
value(10)

?- generate(5).
```

Combining Rules

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).

value(I), value(J) <=> K is I + J, value(K).
```

What's the outcome?

```
?- generate(4).
value(10)

?- generate(5).
value(15)
```

In Slow Motion

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).

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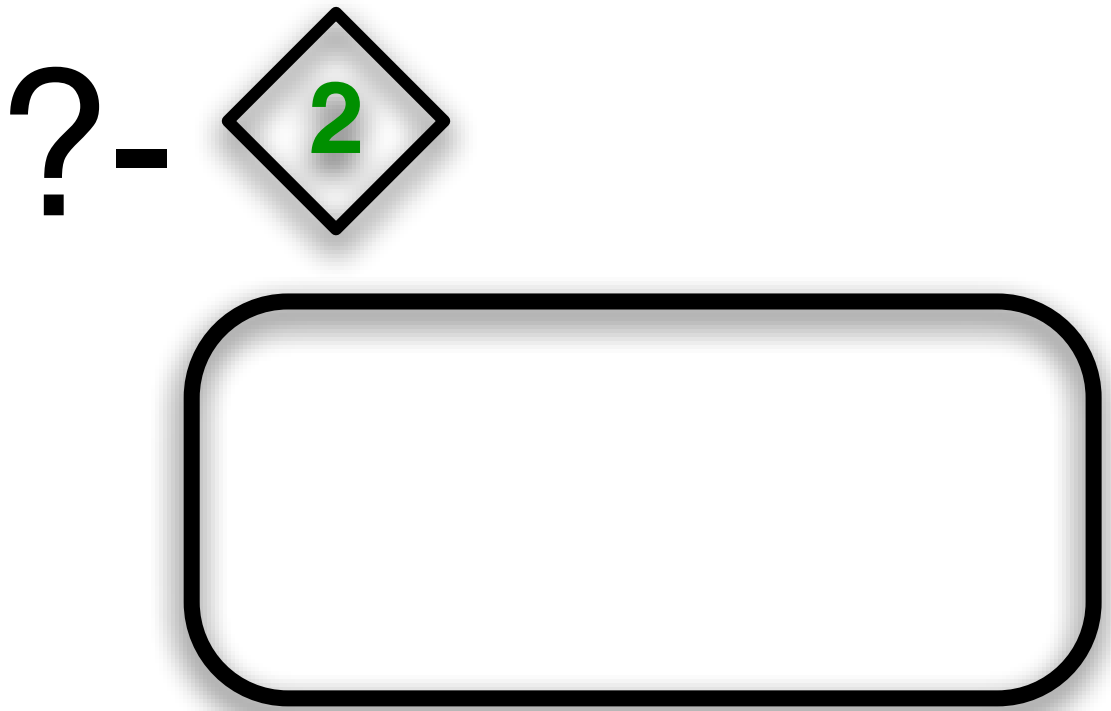
```



In Slow Motion

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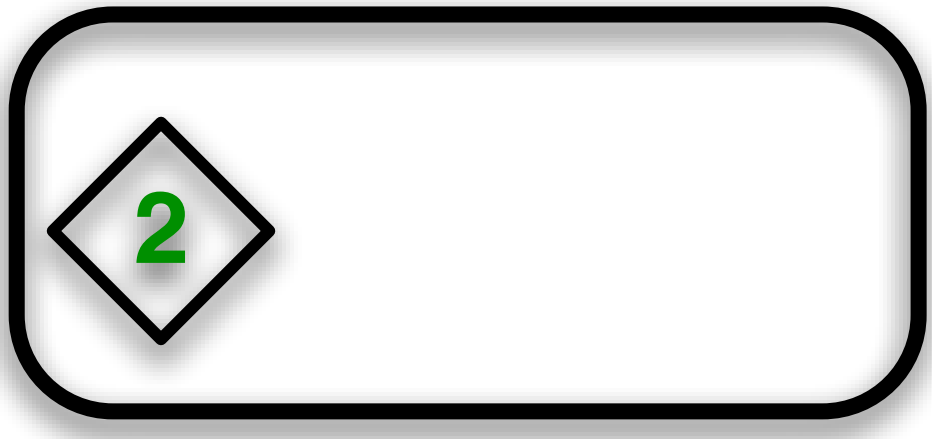
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).

value(I), value(J) <=> K is I + J, value(K).
```

?-



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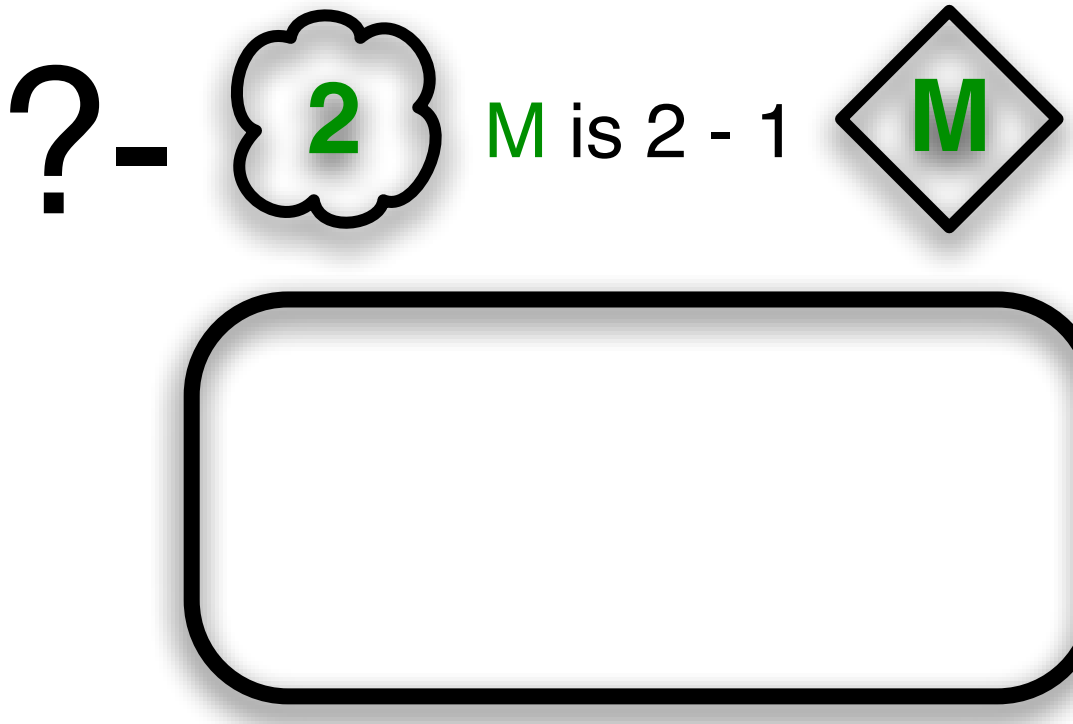

In Slow Motion

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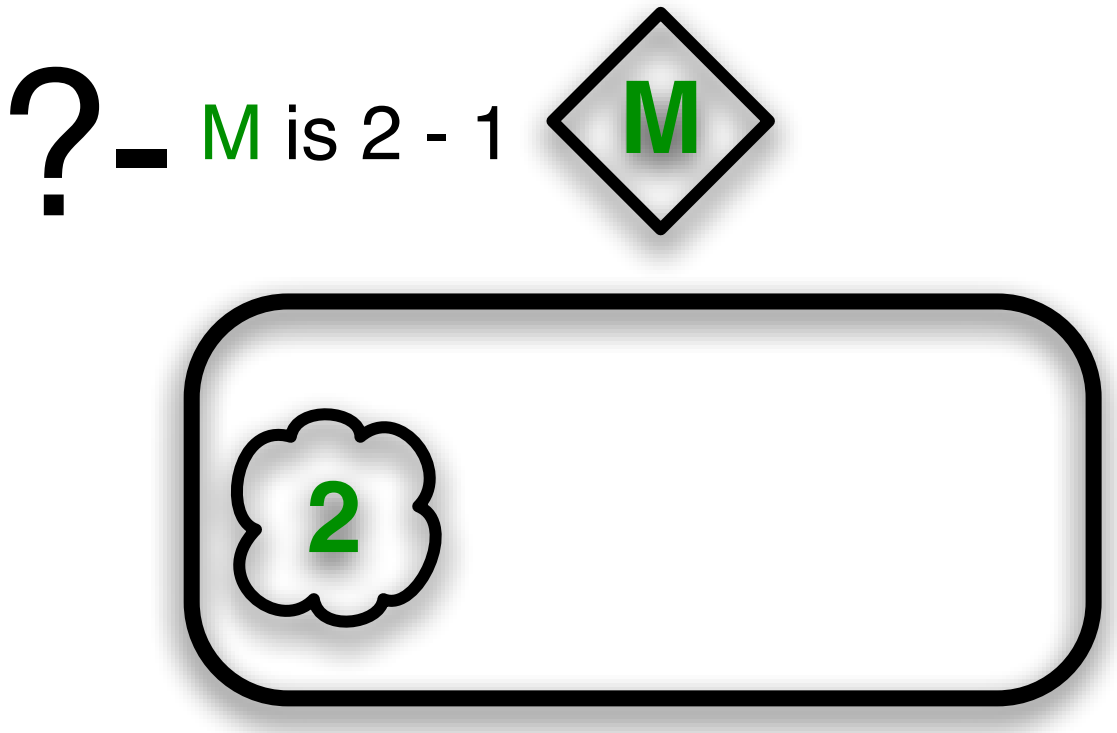

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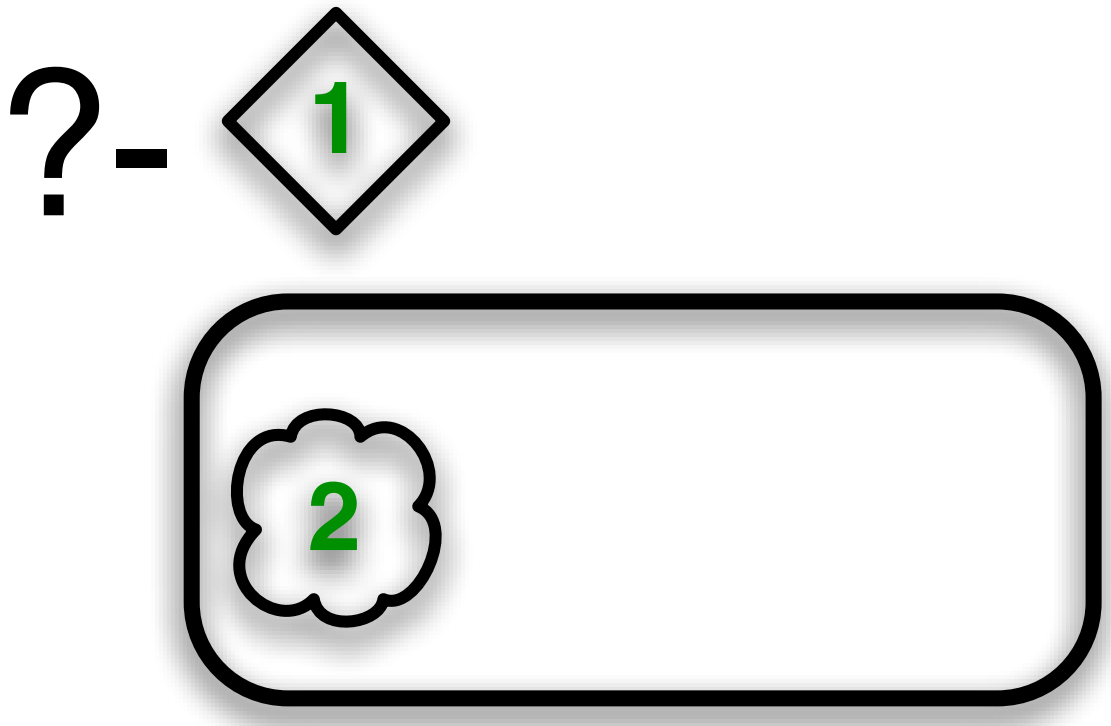

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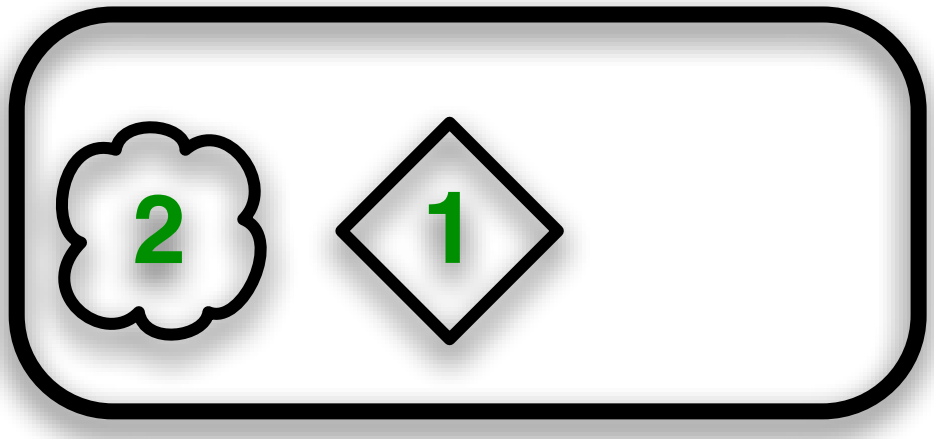
```

?-



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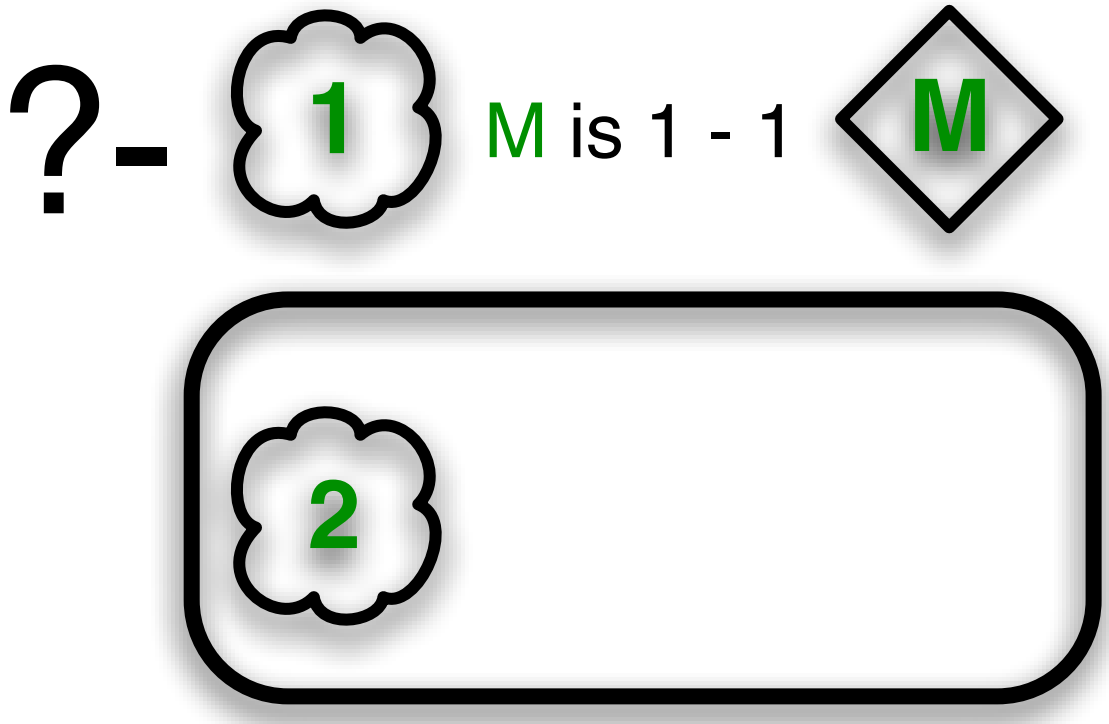

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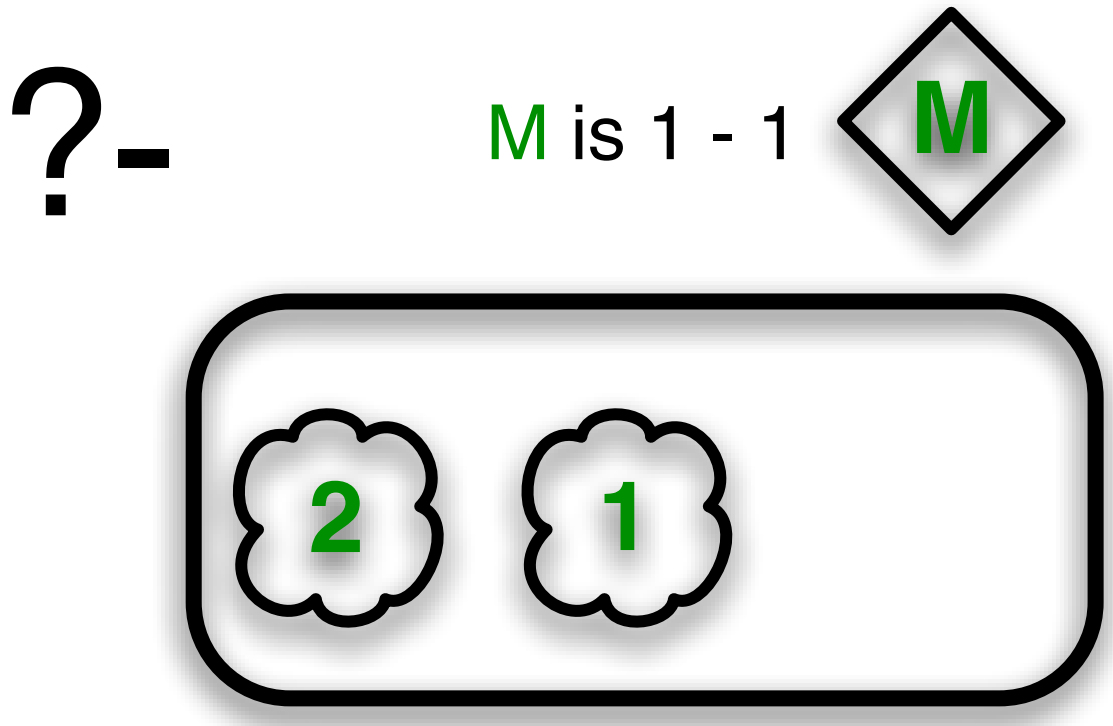

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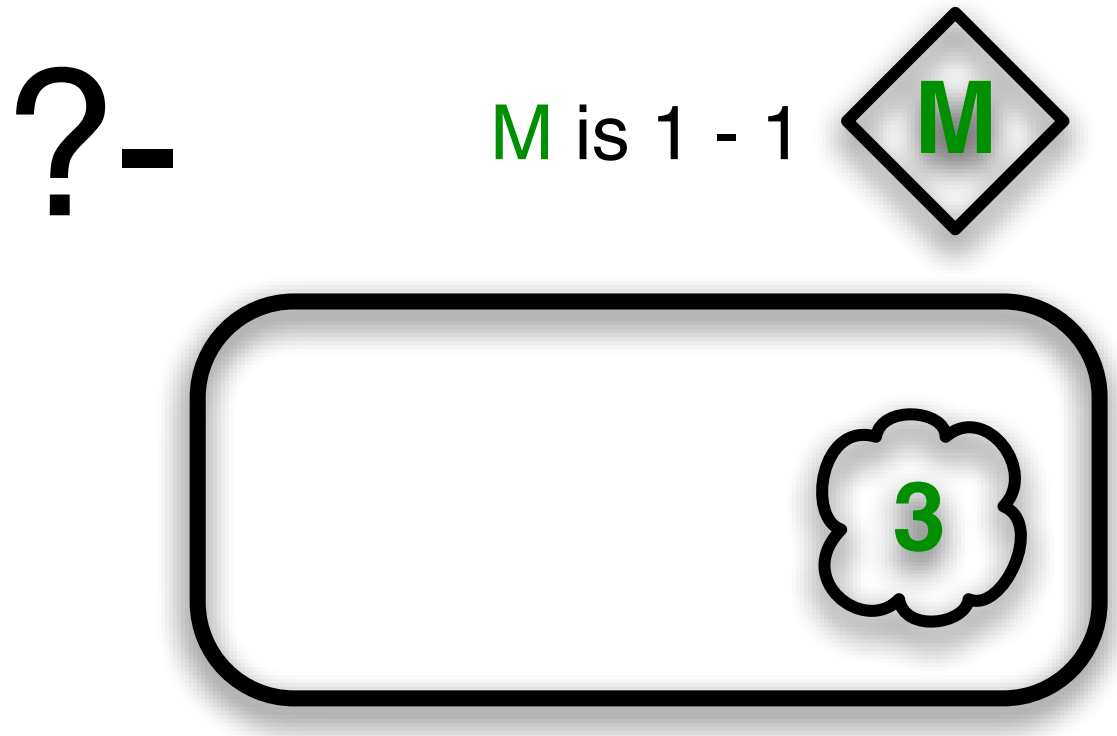

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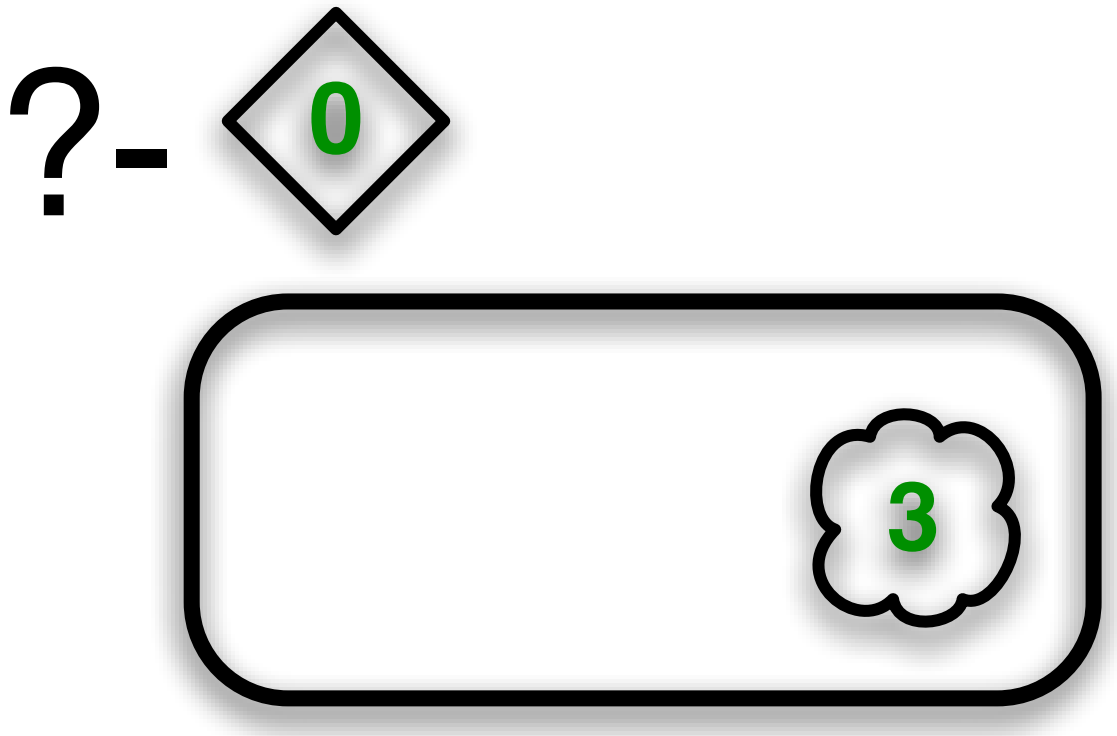

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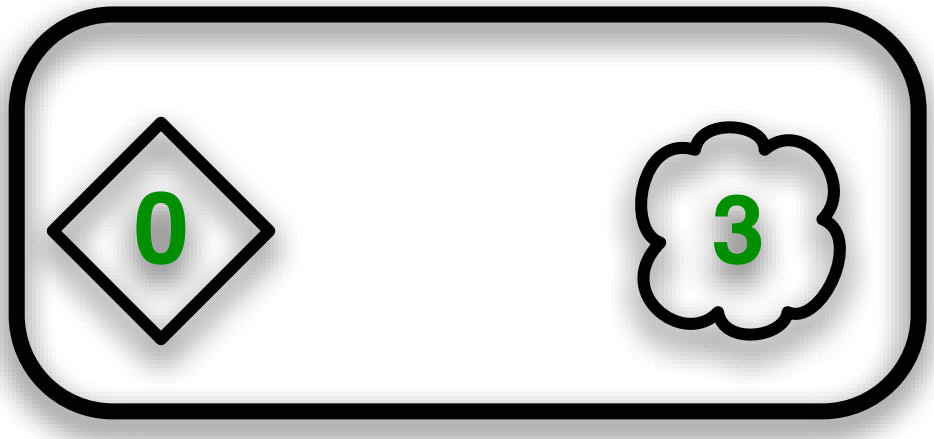

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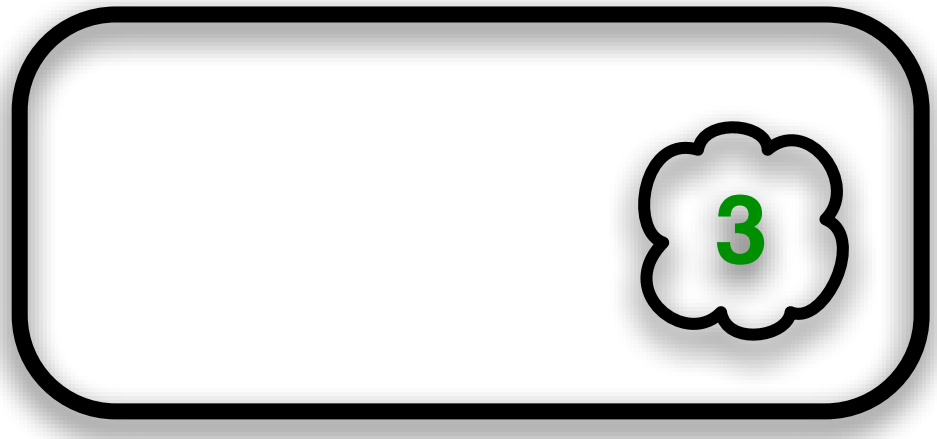

Combining Rules

```
generate(1) <=> true.
generate(N) <=> N > 1 | value(N),
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value(I) \ value(J) <=> J mod I =:= 0 | true.
```

What's the outcome?

```
?- generate(4).
```

Combining Rules

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```
?- generate(4).
value(2)
value(3)
```

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What's the outcome?

```
?- generate(4).
value(2)
value(3)

?- generate(10).
```

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```

What's the outcome?

```
?- generate(4).
value(2)
value(3)

?- generate(10).
value(2)
value(3)
value(5)
value(7)
```

How Does It Work?

```
generate(1) <=> true.
generate(N) <=> N > 1 | value(N),
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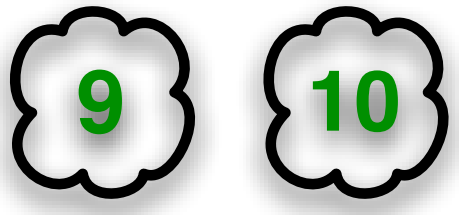

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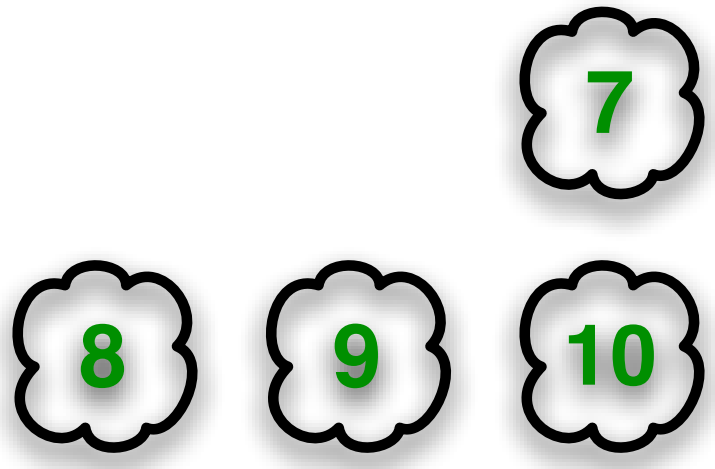

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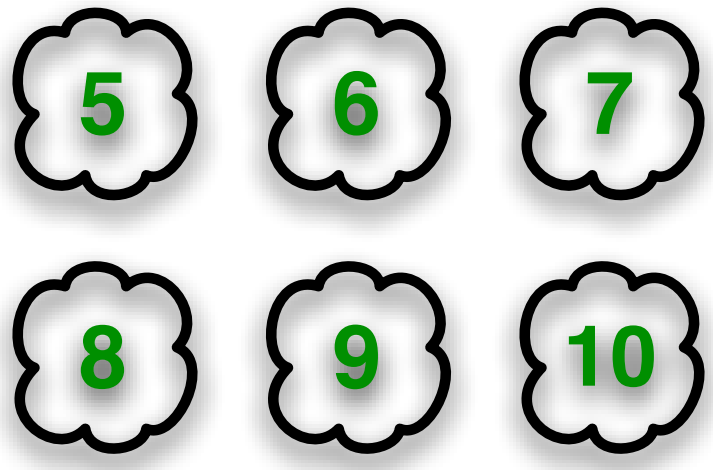

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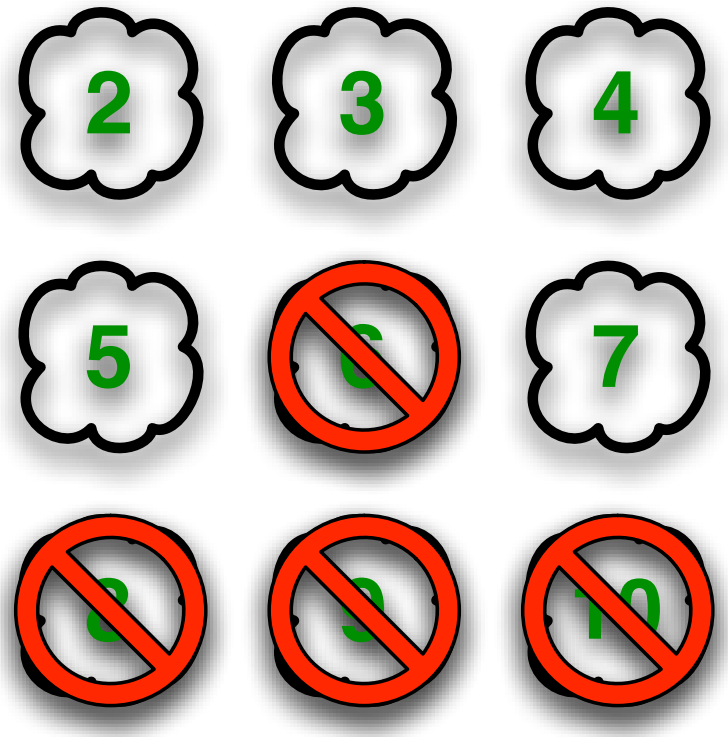

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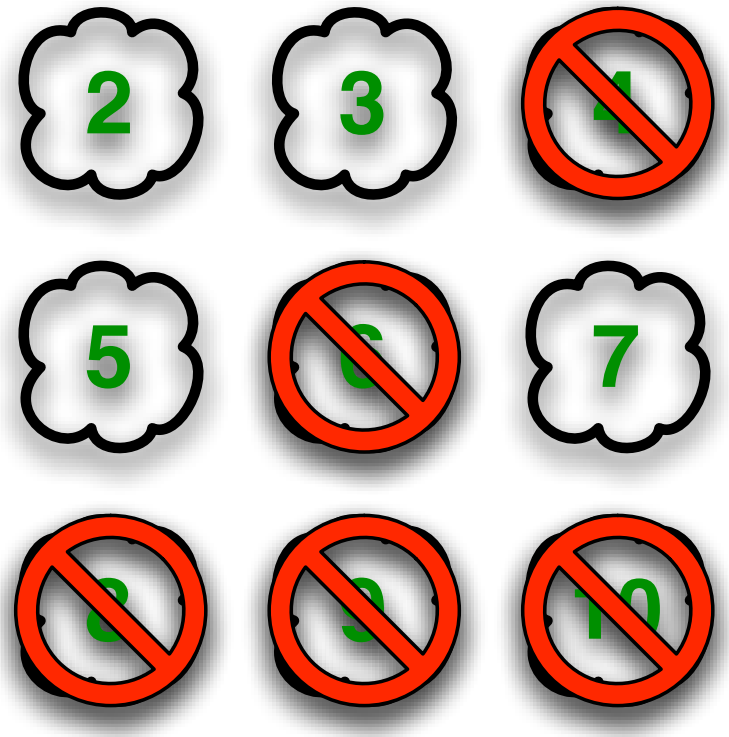

Alternatives?

```
generate(1) <=> true.
generate(N) <=> N > 1 | value(N),
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value(I) \ value(J) <=> J mod I =:= 0 | true.
```

Just added `value(3)`.
What now?

Alternatives?

```
generate(1) <=> true.
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Just added `value(3)`.
What now?

- remove `value(6)` ?

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Just added `value(3)`.
What now?

- remove `value(6)` ?
- remove `value(9)` ?

Alternatives?

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generate(1) <=> true.
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value(I) \ value(J) <=> J mod I =:= 0 | true.
```

Just added `value(3)`.
What now?

- remove `value(6)` ?
- remove `value(9)` ?
- unspecified in CHR!

Alternate Ending

```
generate(1) <=> true.
generate(N) <=> N > 1 | value(N),
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value(I) \ value(J) <=> J mod I =:= 0 | true.
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generate(1) <=> true.
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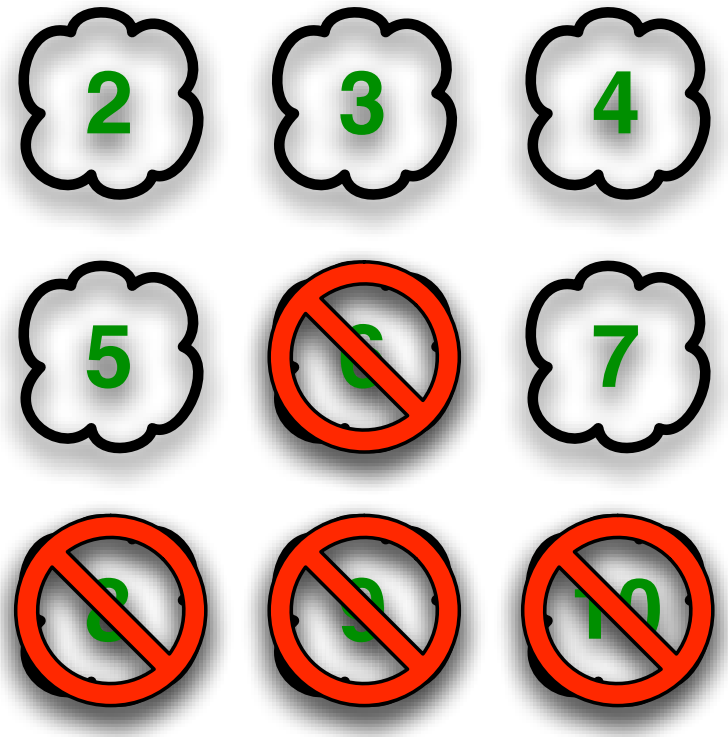

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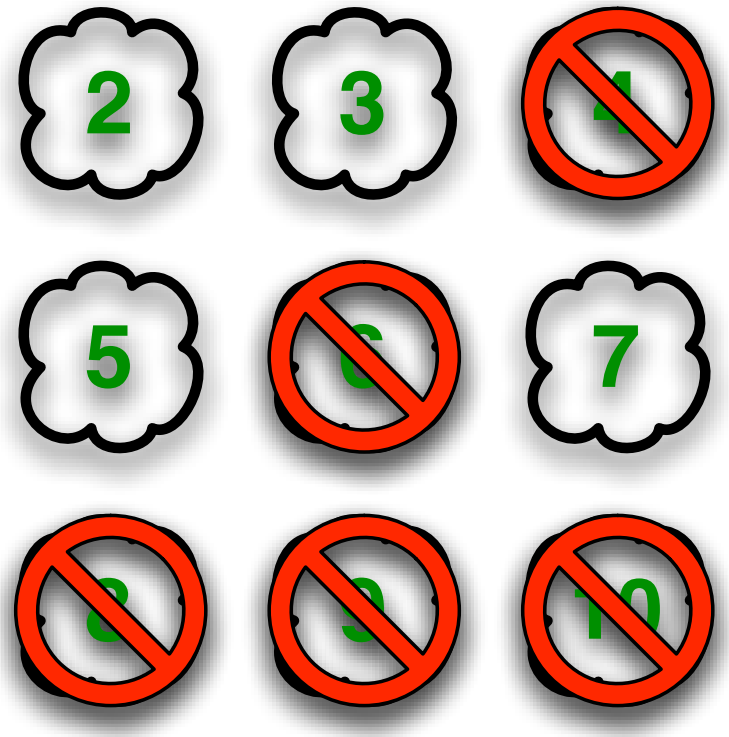

Summary

CHR Programs

- Programs consist of a sequence of rules

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CHR Programs

- Programs consist of a sequence of rules
- Active constraint traverses the rules top-to-bottom to find any that fire
- Rest of query/body waits
- Unspecified which partner is found

Simpagation with $H_k = \emptyset$

Simpagation rule

$$head_k \setminus head_r \mathrel{<=>} guard \mid body\,.$$

Simpagation with $H_k = \emptyset$

Simpagation rule

$head_k \setminus head_r$ <=> $guard \mid body$.

What if $head_k = \emptyset$?

Simpagation with $H_k = \emptyset$

Simpagation rule

$head_k \setminus head_r$ <=> $guard \mid body$.

What if $head_k = \emptyset$?

Simplification rule

$head_r$ <=> $guard \mid body$.

Simpagation with $H_r = \emptyset$

Simpagation rule

$$head_k \setminus head_r \Leftrightarrow guard \mid body\ .$$

Simpagation with $H_r = \emptyset$

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Simpagation with $H_r = \emptyset$

Simpagation rule

$head_k \setminus head_r <=> guard \mid body$.

What if $head_r = \emptyset$?

Propagation rule

$head_k ==> guard \mid body$.

- add *body*
- in the presence of $head_k$
- if *guard* holds

Propagation

```
a, b ==> c.
```

```
?- a, b.
```

Propagation

```
a, b ==> c.
```

```
?- a, b.
a
b
c
```

Propagation

```
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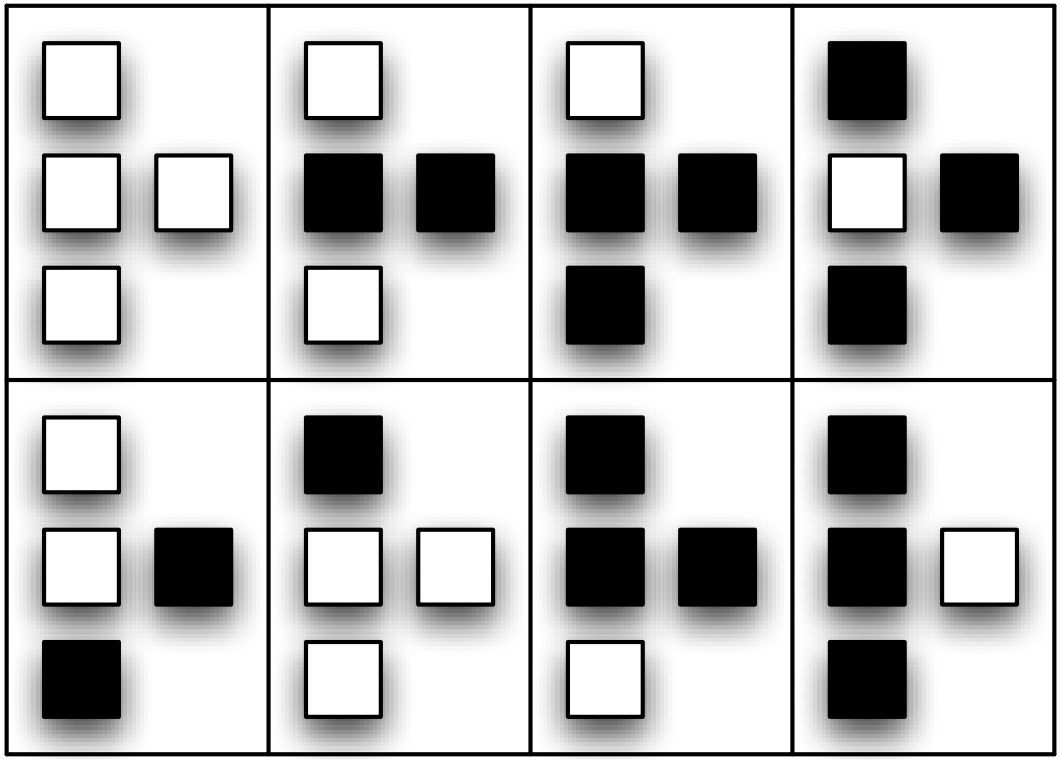
```

Cellular Automaton

Writing Wolfram's celluar automaton Rule 110 in CHR.



Cellular Automaton

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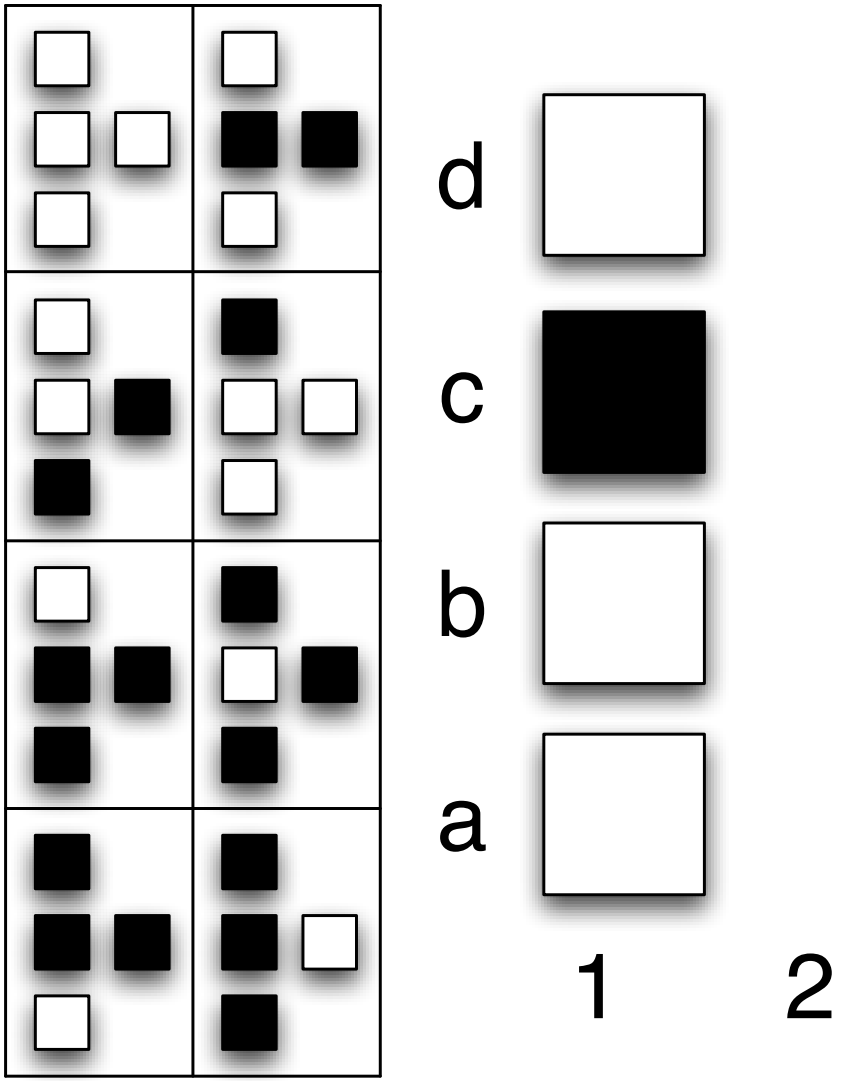

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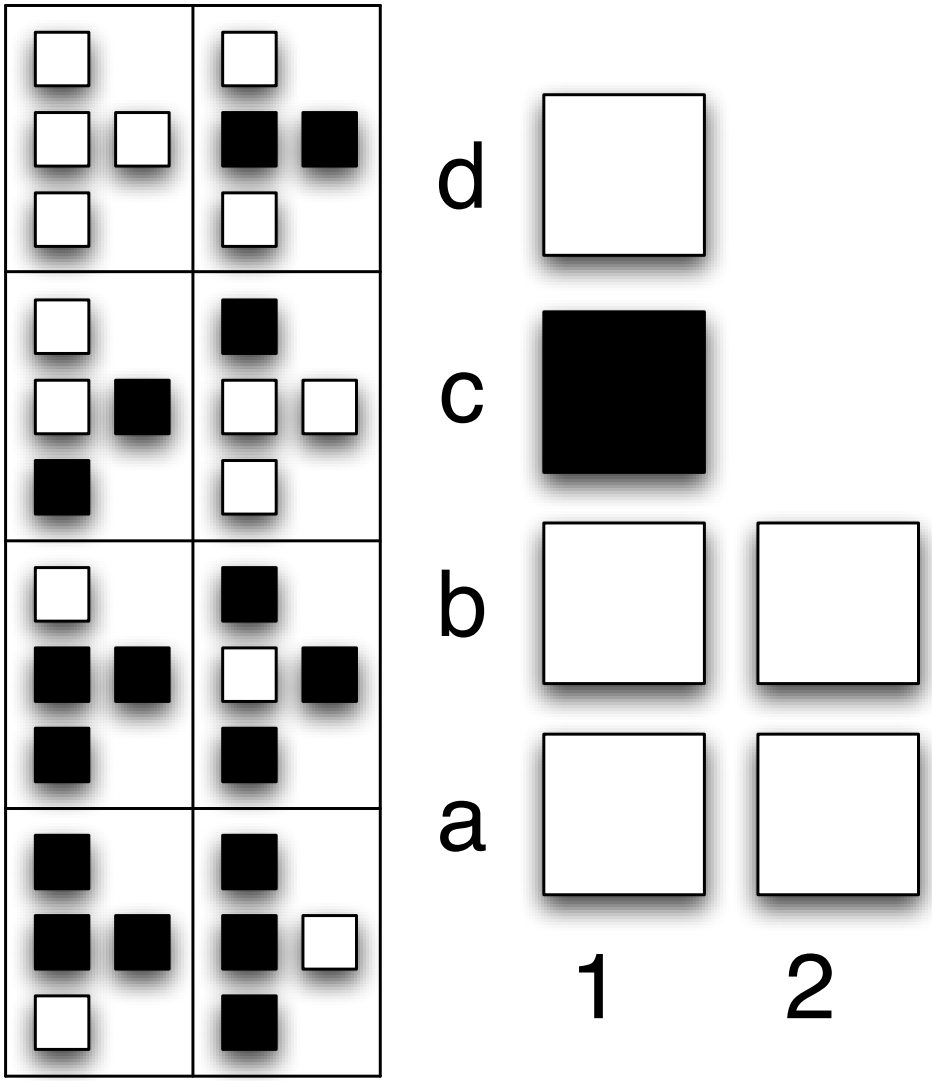

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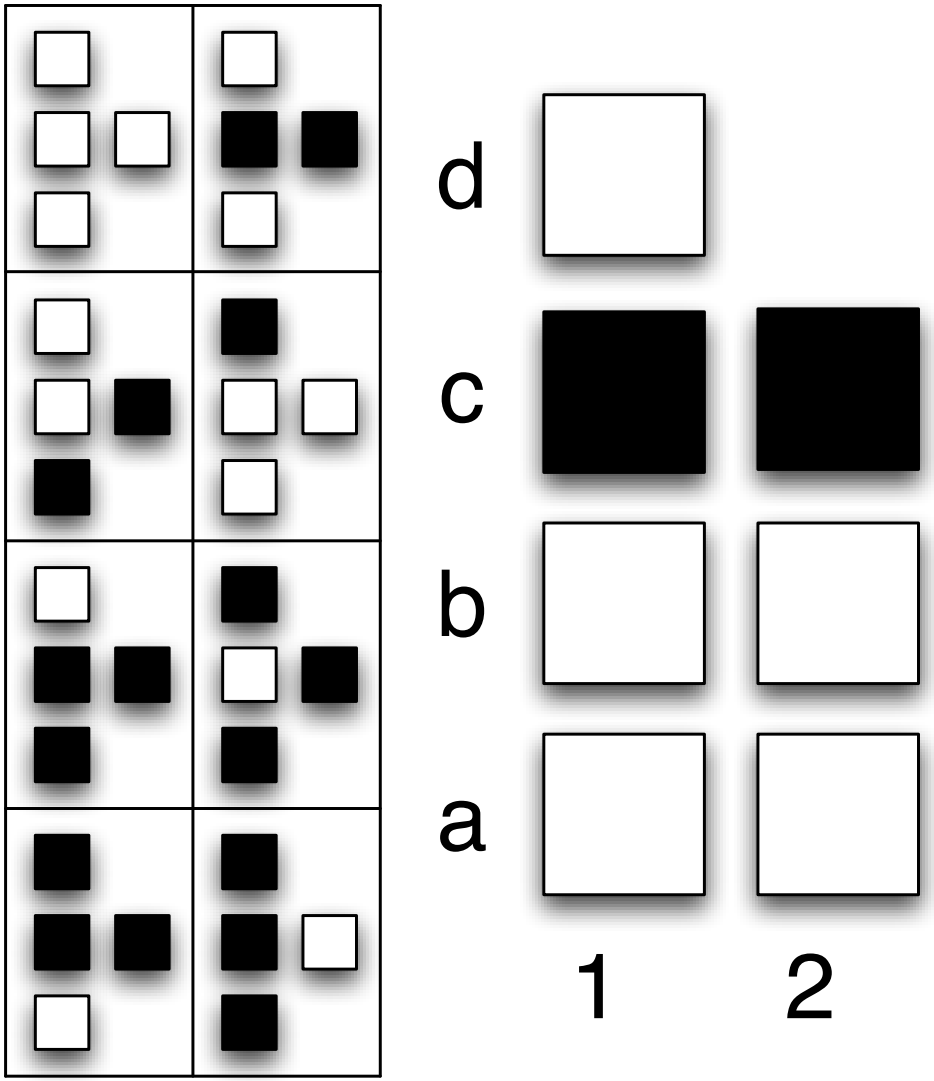

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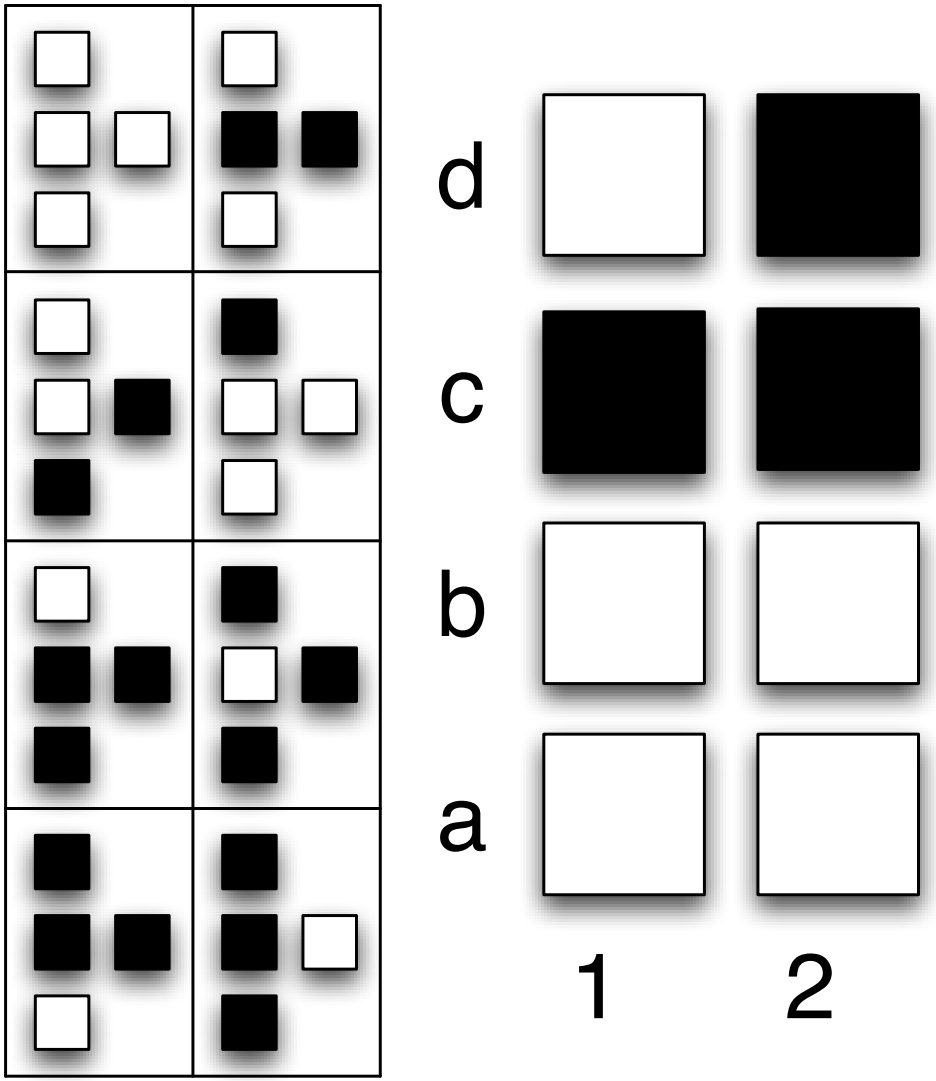

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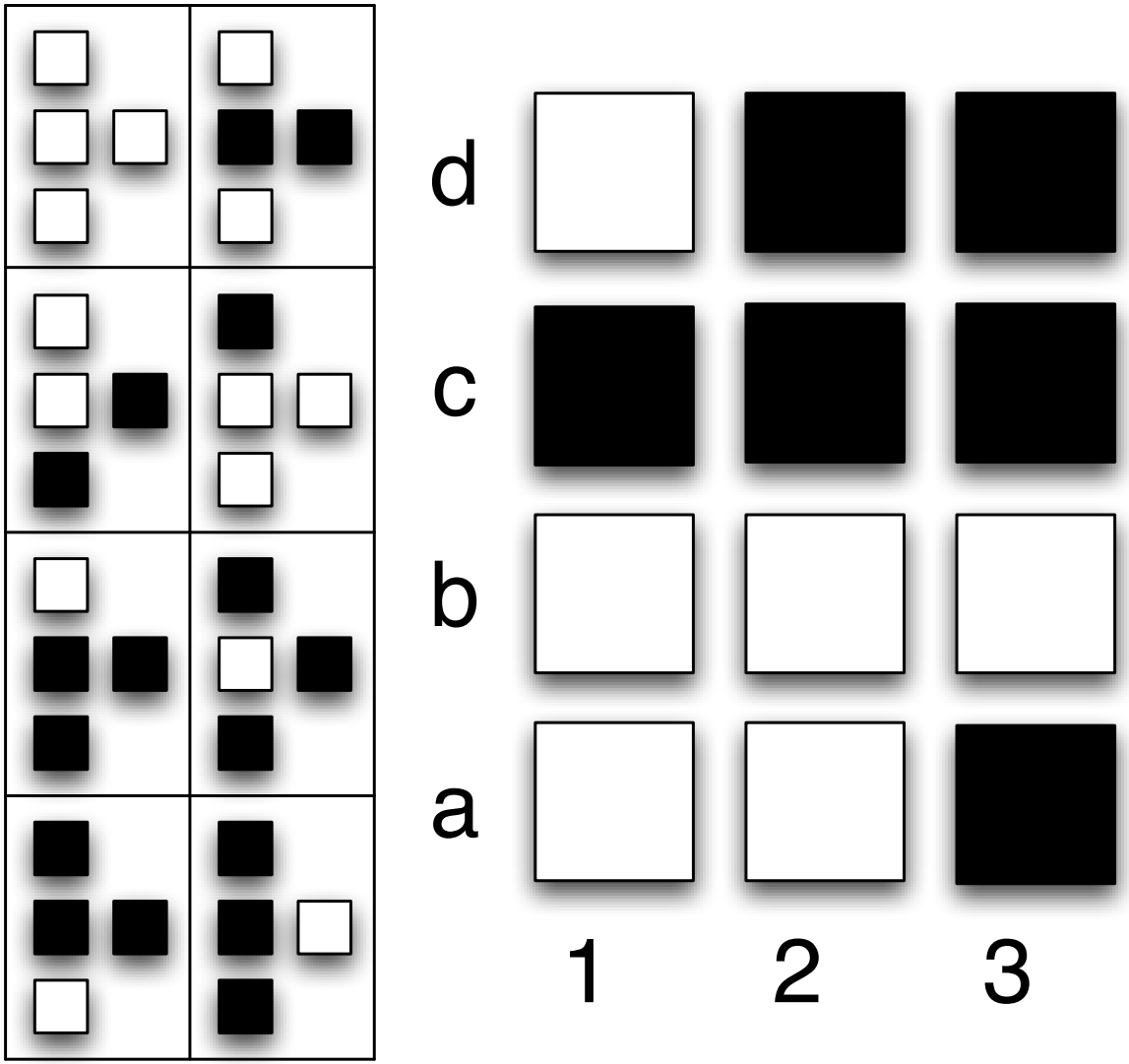

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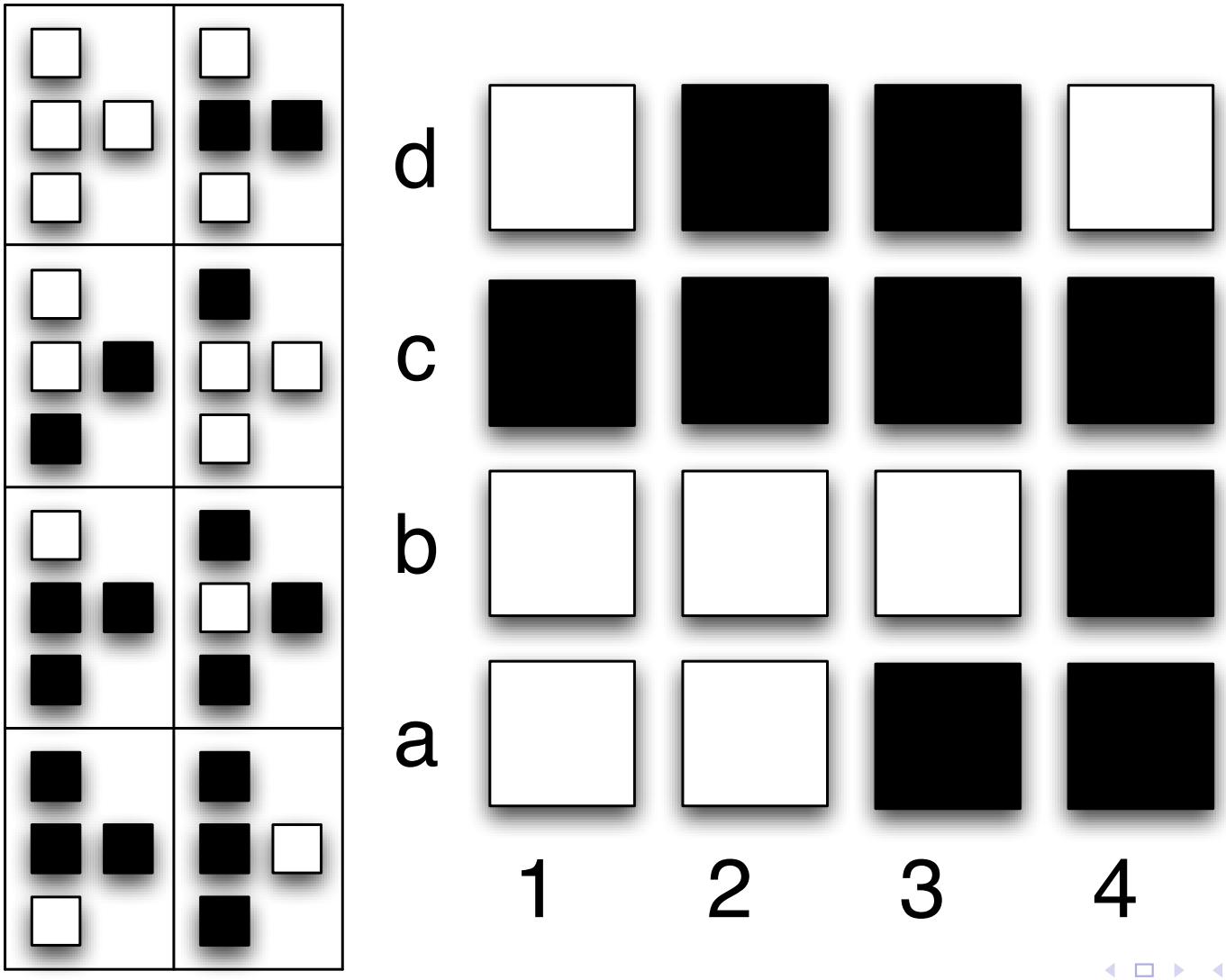

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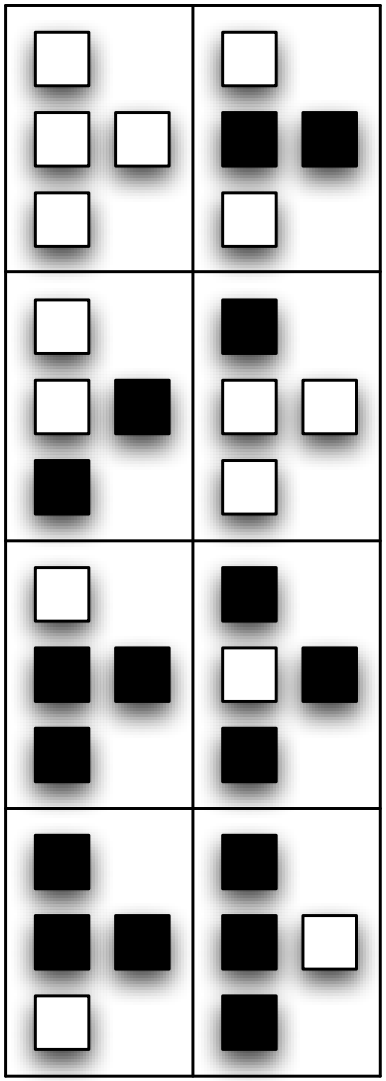

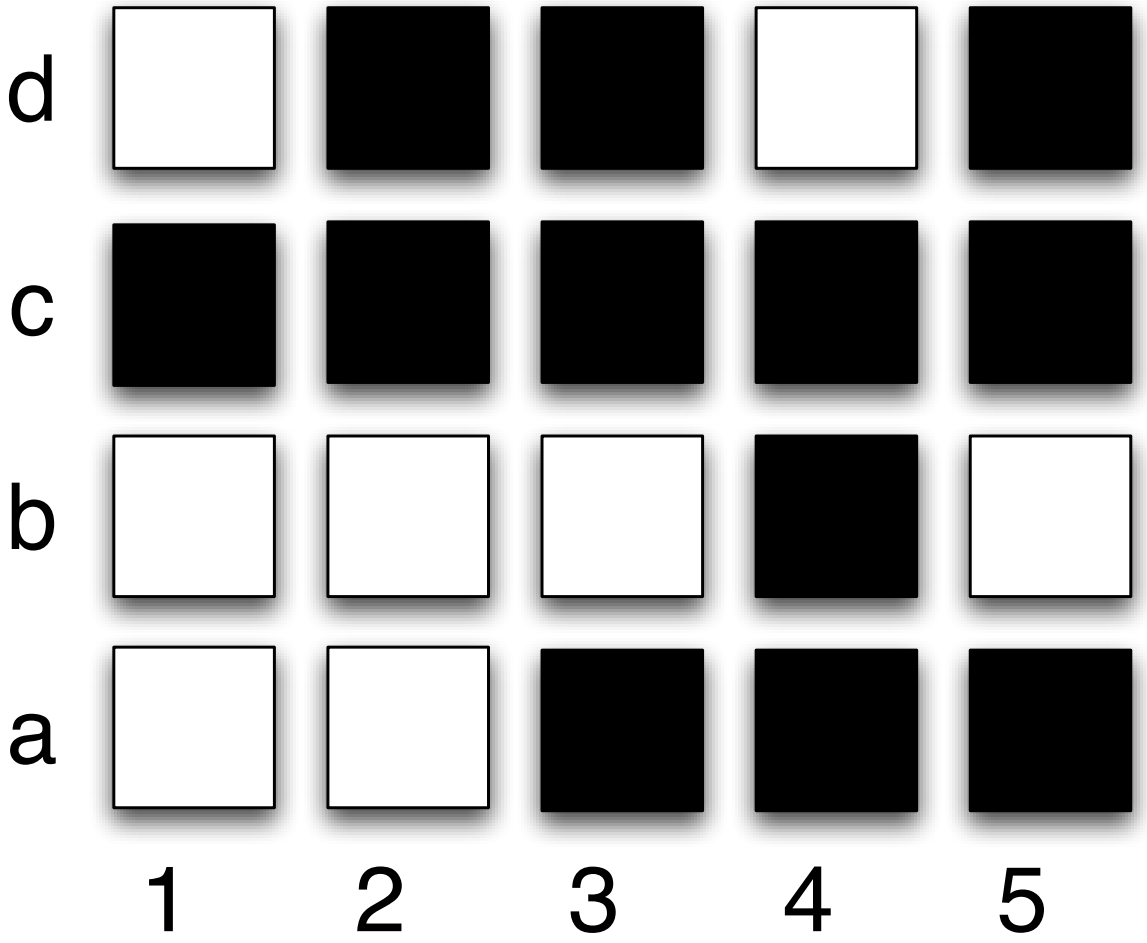

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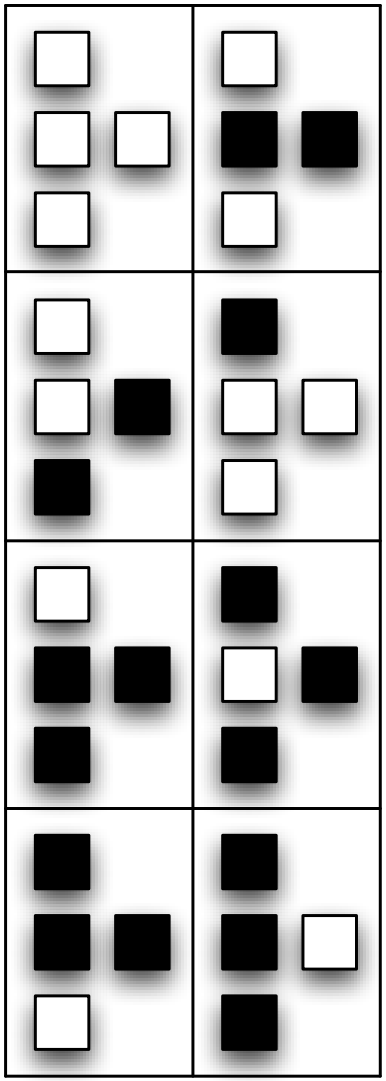
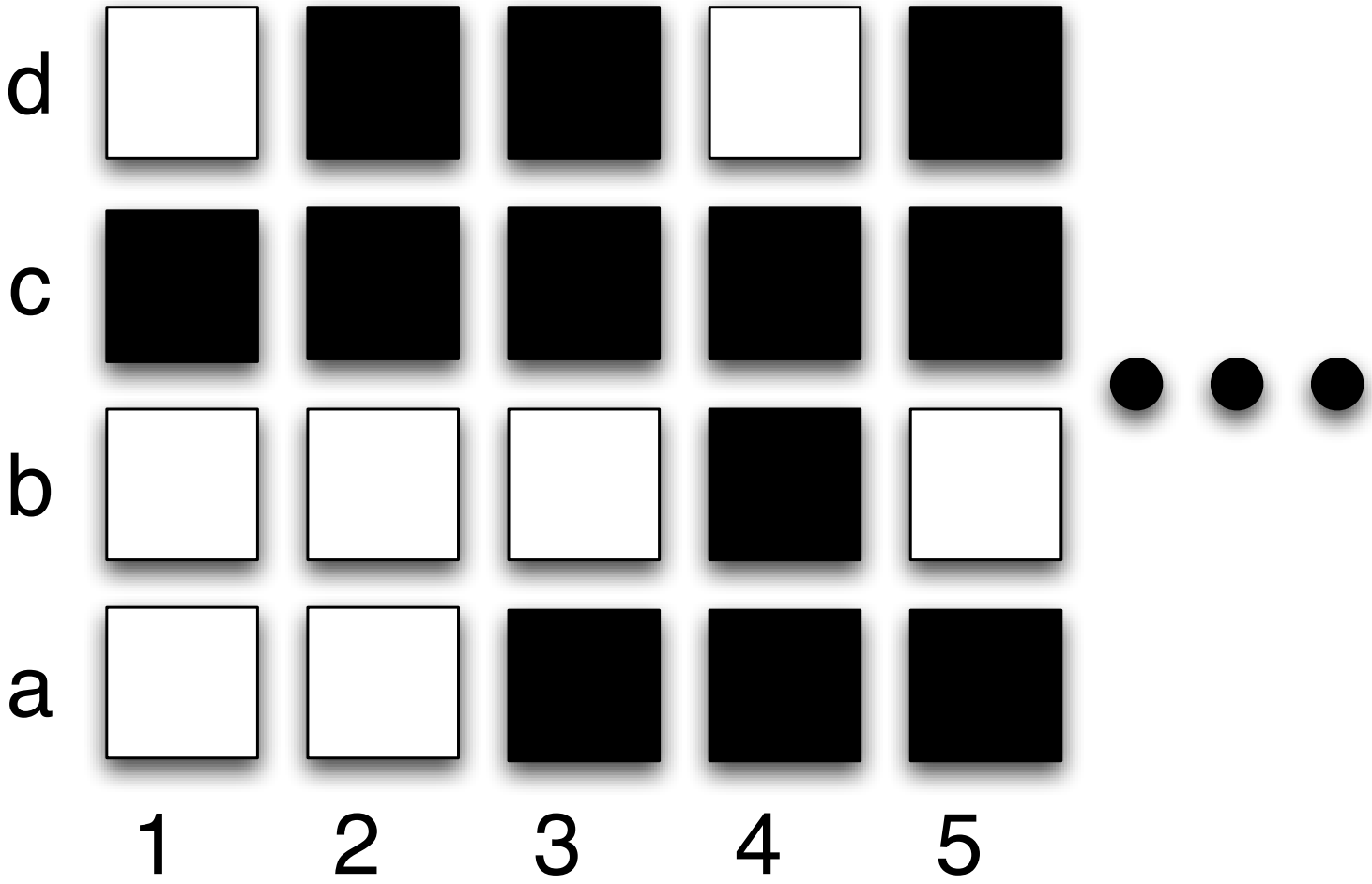
Writing Wolfram's celluar automaton Rule 110 in CHR.





Cellular Automaton

CHR cells

```
% black(Name,Below,Above,Generation).
:- chr_constraint black/4.

% white(Name,Below,Above,Generation).
:- chr_constraint white/4.
```

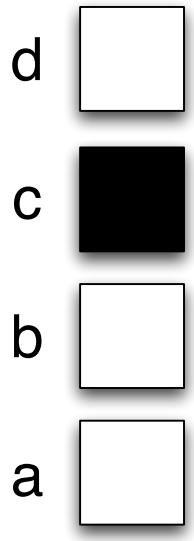
Cellular Automaton

CHR cells

```
% black(Name,Below,Above,Generation).
:- chr_constraint black/4.

% white(Name,Below,Above,Generation).
:- chr_constraint white/4.
```

```
?- white(d,c,a,1),
   black(c,b,d,1),
   white(b,a,c,1),
   white(a,d,b,1).
```



1

Automaton Rule

```
white(C,_,_,G) ,
black(B,A,C,G) ,
white(A,_,_,G)
   ==> G < 10 | NG is G + 1, black(B,A,C,NG).
```

Automaton Rule

```
white(C,_,_,G) ,
black(B,A,C,G) ,
white(A,_,_,G)
  ==> G < 10 | NG is G + 1, black(B,A,C,NG).
```

```
?- white(d,c,a,1),
   black(c,b,d,1),
   white(b,a,c,1),
   white(a,d,b,1).
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Automaton Rule

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?- white(d,c,a,1),
   black(c,b,d,1),
   white(b,a,c,1),
   white(a,d,b,1).
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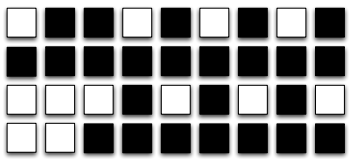

Another Example of Propagation

```
generate(N) ==> value(2).
```

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```
generate(N) ==> value(2).

generate(N), value(I) ==>
        I < N | J is I + 1, value(J).
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```
generate(N) ==> value(2).

generate(N), value(I) ==>
        I < N | J is I + 1, value(J).

value(I) \ value(J) <=> J mod I =:= 0 | true.
```

```
?- generate(10).
```

Another Example of Propagation

```
generate(N) ==> value(2).

generate(N), value(I) ==>
        I < N | J is I + 1, value(J).

value(I) \ value(J) <=> J mod I =:= 0 | true.
```

```
?- generate(10).
value(2)
value(3)
value(5)
value(7)
generate(10)
```

Summary

Simplification Rule: *head* <=> *guard* | *body* .

- replace head with body

Simpagation Rule: $head_k \backslash head_r$ <=> *guard* | *body* .

- replace $head_r$ with *body*
- in the presence of $head_k$

Propagation Rule: *head* ==> *guard* | *body* .

- add *body*
- in the presence of *head*
- fires once for each combination (propagation history)

Biased Coin?

```
coin <=> heads.
coin <=> tails.
```

What's the outcome?

```
?- coin.
```

Biased Coin?

```
coin <=> heads.
coin <=> tails.
```

What's the outcome?

```
?- coin.
heads
```

Biased Coin?

```
coin <=> heads.
coin <=> tails.
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What's the outcome?

```
?- coin.
heads
```

Rules are tried in order!

Coin Flipping in Prolog

CHR	Prolog
`coin <=> heads.`	`p_coin :- heads.`
`coin <=> tails.`	`p_coin :- tails.`

What's the outcome?

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`coin <=> heads.`	`p_coin :- heads.`
`coin <=> tails.`	`p_coin :- tails.`

What's the outcome?

```
?- coin.
heads
```

Coin Flipping in Prolog

CHR

```
coin <=> heads.
coin <=> tails.
```

Prolog

```
p_coin :- heads.
p_coin :- tails.
```

What's the outcome?

```
?- coin.
heads
```

```
?- p_coin.
heads
```

Coin Flipping in Prolog

CHR	Prolog
`coin <=> heads.`	`p_coin :- heads.`
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What's the outcome?

```
?- coin.
heads
```

```
?- p_coin.
heads ; y
```

Coin Flipping in Prolog

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`coin <=> heads.`	`p_coin :- heads.`
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What's the outcome?

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heads
```

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?- p_coin.
heads ; y

tails
```

Coin Flipping in Prolog

CHR	Prolog
`coin <=> heads.`	`p_coin :- heads.`
`coin <=> tails.`	`p_coin :- tails.`

What's the outcome?

```
?- coin.
heads
```

```
?- p_coin.
heads ; y

tails
```

- Prolog: backtracking
- CHR: **committed choice** for simplification rules

Propagation Coin Flipping

```
coin ==> heads.
coin ==> tails.
```

What's the outcome?

```
?- coin.
heads
tails
coin
```

Propagation Coin Flipping

```
coin ==> heads.
coin ==> tails.
```

What's the outcome?

```
?- coin.
heads
tails
coin
```

Rules are applied in sequence.

Propagation Coin Flipping

```
coin ==> heads.
coin <=> tails.
coin ==> side.
```

What's the outcome?

```
?- coin.
heads
tails
```

Propagation Coin Flipping

```
coin ==> heads.
coin <=> tails.
coin ==> side.
```

What's the outcome?

```
?- coin.
heads
tails
```

Rules are applied in sequence, until the active constraint is removed.

Exploiting Rule Order

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

Exploiting Rule Order

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
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```

vs.

```
generate(0) <=> true.
generate(N) <=> value(N),
                M is N - 1, generate(M).
```

Exploiting Rule Order

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

vs.

```
generate(0) <=> true.
generate(N) <=> value(N),
                M is N - 1, generate(M).
```

for queries `?- generate(N).` with $N \geq 0$

Exploiting Rule Order

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

vs.

```
generate(0) <=> true.
generate(N) <=> value(N),
                M is N - 1, generate(M).
```

for queries `?- generate(N).` with $N \geq 0$

Better Style?

Exploiting Rule Order

```
generate(0) <=> true.
generate(N) <=> N > 0 | value(N),
                        M is N - 1, generate(M).
```

vs.

```
generate(0) <=> true.
generate(N) <=> value(N),
                M is N - 1, generate(M).
```

for queries `?- generate(N).` with $N \geq 0$

Better Style?

Compare to:

```
generate(0) :- !.
generate(N) :- value(N), M is N - 1, generate(M).
```

Summary

Rule Order

- rules are tried from top to bottom
- applied in sequence
- until the active constraint is removed
- committed choice: no alternatives explored for simplification rules

Hello, World!

```
c(hello) <=> writeln(hello).
c(world) <=> writeln(world).
```

```
?- c(hello), c(world).
```

Hello, World!

```
c(hello) <=> writeln(hello).
c(world) <=> writeln(world).
```

```
?- c(hello), c(world).
hello
world
```

Hello, World!

```
c(hello) <=> writeln(hello).
c(world) <=> writeln(world).
```

```
?- c(hello), c(world).
hello
world
?- c(X).
```

Hello, World!

```
c(hello) <=> writeln(hello).
c(world) <=> writeln(world).
```

```
?- c(hello), c(world).
hello
world
?- c(X).
c(X)
```

Hello, World!

```
c(hello) <=> writeln(hello).
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```
?- c(hello), c(world).
hello
world
?- c(X).
c(X)
?- c(X), X = hello.
```

Hello, World!

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c(hello) <=> writeln(hello).
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```
?- c(hello), c(world).
hello
world
?- c(X).
c(X)
?- c(X), X = hello.
hello
```

Hello, World!

```
c(hello) <=> writeln(hello).
c(world) <=> writeln(world).
```

```
?- c(hello), c(world).
hello
world
?- c(X).
c(X)
?- c(X), X = hello.
hello
?- c(X), c(world), X = hello.
```

Hello, World!

```
c(hello) <=> writeln(hello).
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```

```
?- c(hello), c(world).
hello
world
?- c(X).
c(X)
?- c(X), X = hello.
hello
?- c(X), c(world), X = hello.
world
hello
X = hello
```

Reactivation

- constraints suspend in the constraint store
- unification reactivates suspended constraints

A Constraint: Inequality

```
neq(X,X) <=> fail.
neq(X,Y) <=> X \= Y | true.
```

```
?- neq(a,a).
```

A Constraint: Inequality

```
neq(X,X) <=> fail.
neq(X,Y) <=> X \= Y | true.
```

```
?- neq(a,a).
No
```

A Constraint: Inequality

```
neq(X,X) <=> fail.
neq(X,Y) <=> X \= Y | true.
```

```
?- neq(a,a).
No

?- neq(a,b).
```

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No

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Yes
```

A Constraint: Inequality

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neq(X,X) <=> fail.
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```
?- neq(a,a).
No

?- neq(a,b).
Yes

?- neq(A,B).
```

A Constraint: Inequality

```
neq(X,X) <=> fail.
neq(X,Y) <=> X \= Y | true.
```

```
?- neq(A,B), A = B.
```

```
?- neq(a,a).
No

?- neq(a,b).
Yes

?- neq(A,B).
neq(A,B)
```

A Constraint: Inequality

```
neq(X,X) <=> fail.
neq(X,Y) <=> X \= Y | true.
```

```
?- neq(a,a).
No

?- neq(a,b).
Yes

?- neq(A,B).
neq(A,B)
```

```
?- neq(A,B), A = B.
No
```

A Constraint: Inequality

```
neq(X,X) <=> fail.
neq(X,Y) <=> X \= Y | true.
```

```
?- neq(a,a).
No

?- neq(a,b).
Yes

?- neq(A,B).
neq(A,B)
```

```
?- neq(A,B), A = B.
No

?- neq(A,B), A = a, B = a.
```

A Constraint: Inequality

```
neq(X,X) <=> fail.
neq(X,Y) <=> X \= Y | true.
```

```
?- neq(a,a).
No

?- neq(a,b).
Yes

?- neq(A,B).
neq(A,B)
```

```
?- neq(A,B), A = B.
No

?- neq(A,B), A = a, B = a.
No
```

A Constraint: Inequality

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?- neq(a,a).
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neq(A,B)
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A Constraint: Inequality

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neq(X,X) <=> fail.
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neq(A,B)
```

```
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Yes
```

A Constraint: Inequality

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neq(X,X) <=> fail.
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No

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Yes

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neq(A,B)
```

```
?- neq(A,B), A = B.
No

?- neq(A,B), A = a, B = a.
No

?- neq(A,B), A = a, B = b.
Yes

?- neq(A,B), A = f(C), B = f(D).
```

A Constraint: Inequality

```
neq(X,X) <=> fail.
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```

```
?- neq(a,a).
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?- neq(a,b).
Yes

?- neq(A,B).
neq(A,B)
```

```
?- neq(A,B), A = B.
No

?- neq(A,B), A = a, B = a.
No

?- neq(A,B), A = a, B = b.
Yes

?- neq(A,B), A = f(C), B = f(D).
neq(f(C),f(D))
```

Using Reactivation for Finite Domains

```
domain(X,L)    <=> ground(X) | memberchk(X,L).
domain(X,[])   <=> fail.
domain(X,[V]) <=> X = V.
domain(X,L1), domain(X,L2) <=>
        intersection(L1,L2,L), domain(X,L).
intersection(L1,L2,L3) :- ...
```

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```

```
?- domain(X,[dog,fox,horse,snails,zebra]).
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domain(X,[dog,fox,horse,snail,zebra])
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intersection(L1,L2,L3) :- ...
```

```
?- domain(X,[dog,fox,horse,snails,zebra]).
domain(X,[dog,fox,horse,snail,zebra])
?- domain(X,[d,f,h,s,z]), X = z.
```

Using Reactivation for Finite Domains

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domain(X,L)    <=> ground(X) | memberchk(X,L).
domain(X,[])   <=> fail.
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intersection(L1,L2,L3) :- ...
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?- domain(X,[dog,fox,horse,snails,zebra]).
domain(X,[dog,fox,horse,snail,zebra])
?- domain(X,[d,f,h,s,z]), X = z.
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```

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?- domain(X,[dog,fox,horse,snails,zebra]).
domain(X,[dog,fox,horse,snail,zebra])
?- domain(X,[d,f,h,s,z]), X = z.
X = z
?- domain(X,[d,f,h,s,z]), domain(X,[c,s,z]).
```

Using Reactivation for Finite Domains

```
domain(X,L)   <=> ground(X) | memberchk(X,L).
domain(X,[])  <=> fail.
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intersection(L1,L2,L3) :- ...
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```
?- domain(X,[dog,fox,horse,snails,zebra]).
domain(X,[dog,fox,horse,snail,zebra])
?- domain(X,[d,f,h,s,z]), X = z.
X = z
?- domain(X,[d,f,h,s,z]), domain(X,[c,s,z]).
domain(X,[s,z])
```

Using Reactivation for Finite Domains

```
domain(X,L)   <=> ground(X) | memberchk(X,L).
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intersection(L1,L2,L3) :- ...
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```
?- domain(X,[dog,fox,horse,snails,zebra]).
domain(X,[dog,fox,horse,snail,zebra])
?- domain(X,[d,f,h,s,z]), X = z.
X = z
?- domain(X,[d,f,h,s,z]), domain(X,[c,s,z]).
domain(X,[s,z])
?- domain(X,[d,f,h,s,z]), domain(X,[c,z]).
```

Using Reactivation for Finite Domains

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?- domain(X,[dog,fox,horse,snails,zebra]).
domain(X,[dog,fox,horse,snail,zebra])
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X = z
```

Summary

Reactivation

- rules may not fire because of lack of instantiation

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- upon instantiation, the rule may now fire

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Reactivation

- rules may not fire because of lack of instantiation
- upon instantiation, the rule may now fire
- useful for implementing constraint solvers

Summary: CHR vs Prolog

	Prolog	CHR
heads	1	≥ 1
rule selection	unification	matching & guard
different rules	alternatives/backtracking	try all in sequence
no rule	failure	delay

CHR and Backtracking

```
p :- a.          a <=> c1.          b <=> d1.
p :- b.          a <=> c2.          b <=> d2.
```

```
?- p.
c1
```

CHR and Backtracking

```
p :- a.          a <=> c1.          b <=> d1.
p :- b.          a <=> c2.          b <=> d2.
```

```
?- p.
c1   ; y
d1
```

CHR and Backtracking

```
p :- a.          a <=> c1.          b <=> d1.
p :- b.          a <=> c2.          b <=> d2.
```

```
?- p.
c1   ; y
d1
```

- Prolog creates choicepoints
- CHR does not
- Prolog backtracking undoes CHR changes

Backtracking for Labeling

```
domain(X,L)    <=> ground(X) | memberchk(X,L).
domain(X,[])   <=> fail.
domain(X,[V]) <=> X = V.
domain(X,L1), domain(X,L2) <=>
        intersection(L1,L2,L), domain(X,L).

split(X) <=> ground(X) | true.
split(X), domain(X,[V|Vs]) <=> X = V ; domain(X,Vs).
```

```
?- domain(X,[d,z]), split(X).
```

Backtracking for Labeling

```
domain(X,L)    <=> ground(X) | memberchk(X,L).
domain(X,[])   <=> fail.
domain(X,[V]) <=> X = V.
domain(X,L1), domain(X,L2) <=>
        intersection(L1,L2,L), domain(X,L).

split(X) <=> ground(X) | true.
split(X), domain(X,[V|Vs]) <=> X = V ; domain(X,Vs).
```

```
?- domain(X,[d,z]), split(X).
X = d  ; y
X = z
```

Backtracking for Labeling

```
domain(X,L)    <=> ground(X) | memberchk(X,L).
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domain(X,L1), domain(X,L2) <=>
        intersection(L1,L2,L), domain(X,L).

split(X) <=> ground(X) | true.
split(X), domain(X,[V|Vs]) <=> X = V ; domain(X,Vs).
```

```
?- domain(X,[d,z]), split(X).
X = d  ; y
X = z

?- domain(X,[d,f,z]), split(X).
```

Backtracking for Labeling

```
domain(X,L)    <=> ground(X) | memberchk(X,L).
domain(X,[])   <=> fail.
domain(X,[V]) <=> X = V.
domain(X,L1), domain(X,L2) <=>
        intersection(L1,L2,L), domain(X,L).

split(X) <=> ground(X) | true.
split(X), domain(X,[V|Vs]) <=> X = V ; domain(X,Vs).
```

```
?- domain(X,[d,z]), split(X).
X = d  ; y
X = z

?- domain(X,[d,f,z]), split(X).
X = d  ; y
domain(X,[f,z])
```

Backtracking for Labeling

```
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indomain(X) <=> ground(X) | true.
% indomain(X), domain(X,[V1,...,Vn]) <=> X = V1;...;X = Vn.
```

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```
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indomain(X), domain(X,Vs) <=> member(X,Vs).
```

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```

```
?- domain(X,[d,f,z]), indomain(X).
X = d  ; y
X = f  ; y
X = z
```

Summary

You should now have an understanding of:

Summary

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- how CHR works.

Summary

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- how CHR works.
- how CHR differs from Prolog.

Summary

You should now have an understanding of:

- how CHR works.
- how CHR differs from Prolog.
- that CHR is not exclusively about constraints .

History: Programming Highlights

1991 **CHR** is born, Thom Frühwirth

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1995 Christian Holzbaur implements **CHR(SICStus)**

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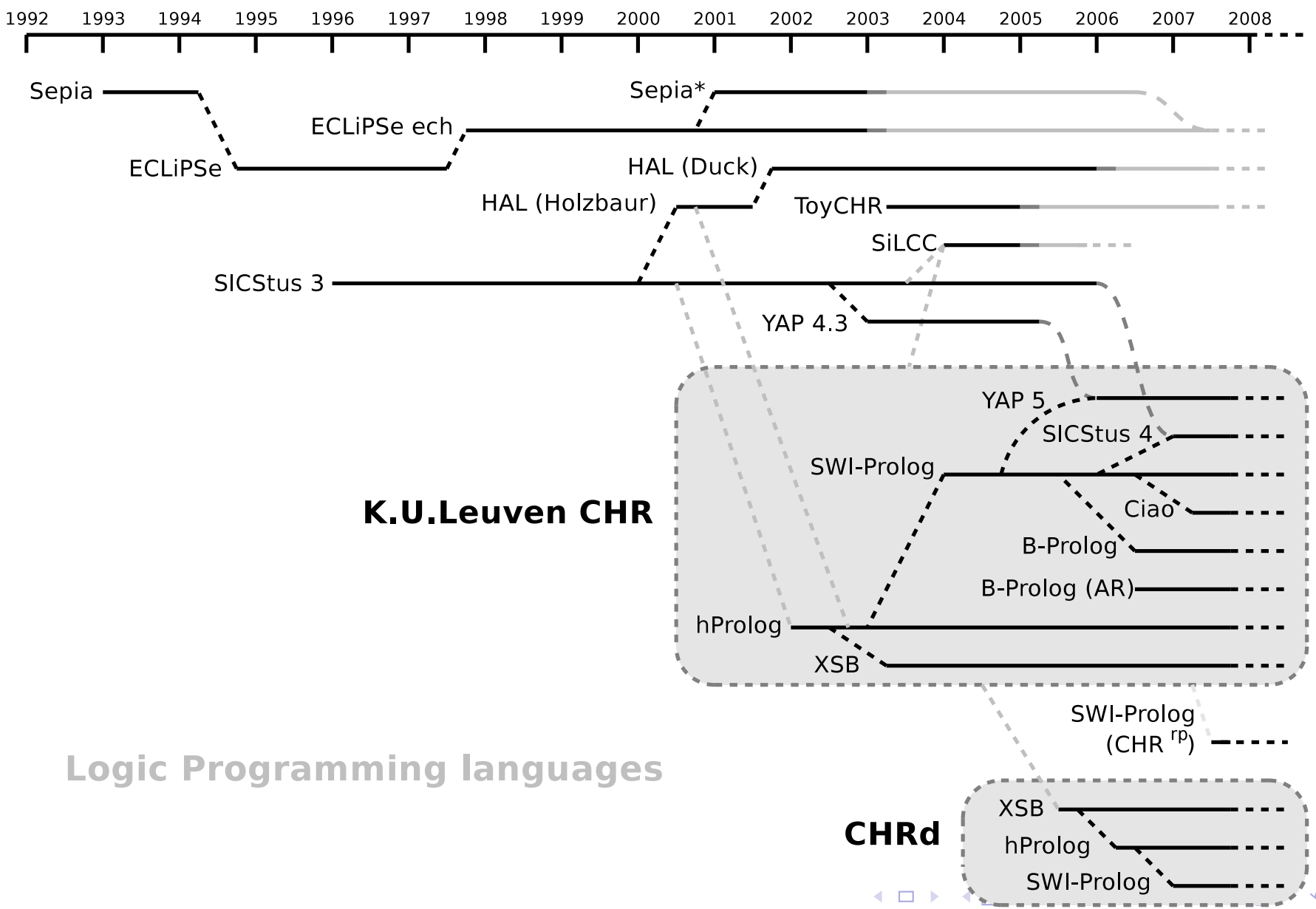
2007	first **concurrent system**, Sulzmann & Lam

CHR Systems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2004
2005
2006
2007
2008
Sepia
ECLiPSe
ECLiPSe ech
Sepia*
HAL (Duck)
HAL (Holzbaur)
ToyCHR
SiLCC
SICStus 3
YAP 4.3
K.U.Leuven CHR
YAP 5
SICStus 4
SWI-Prolog
Ciao
B-Prolog
B-Prolog (AR)
hProlog
XSB
SWI-Prolog
(CHR rp)
Logic Programming languages
CHRd
XSB
hProlog
SWI-Prolog

CHR Programs

Thom Frühwirth, with his students:

- lots of example programs, constraint solvers and others

Thom Frühwirth & Tom Schrijvers,

- union-find

Jon Sneyers,

- Fibonacci heaps and Dijkstra's shortest path
- Hopcroft's DFA minimization
- Turing and RAM machine simulators

Henning Christiansen,

- meta-programming in CHR

...

Declarative Semantics

- classical first-order logic (Frühwirth)
- linear logic (Betz)

CHR Theory & Applications

Declarative Semantics

- classical first-order logic (Frühwirth)
- linear logic (Betz)

Rewriting Properties

- confluence (Frühwirth;Duck;Haemmerle)
- completion (Frühwirth&Abdennadher)
- termination (Frühwirth;Voets&Pilozzi)
- complexity (Frühwirth;De Koninck)

CHR Theory & Applications

Declarative Semantics

- classical first-order logic (Frühwirth)
- linear logic (Betz)

Rewriting Properties

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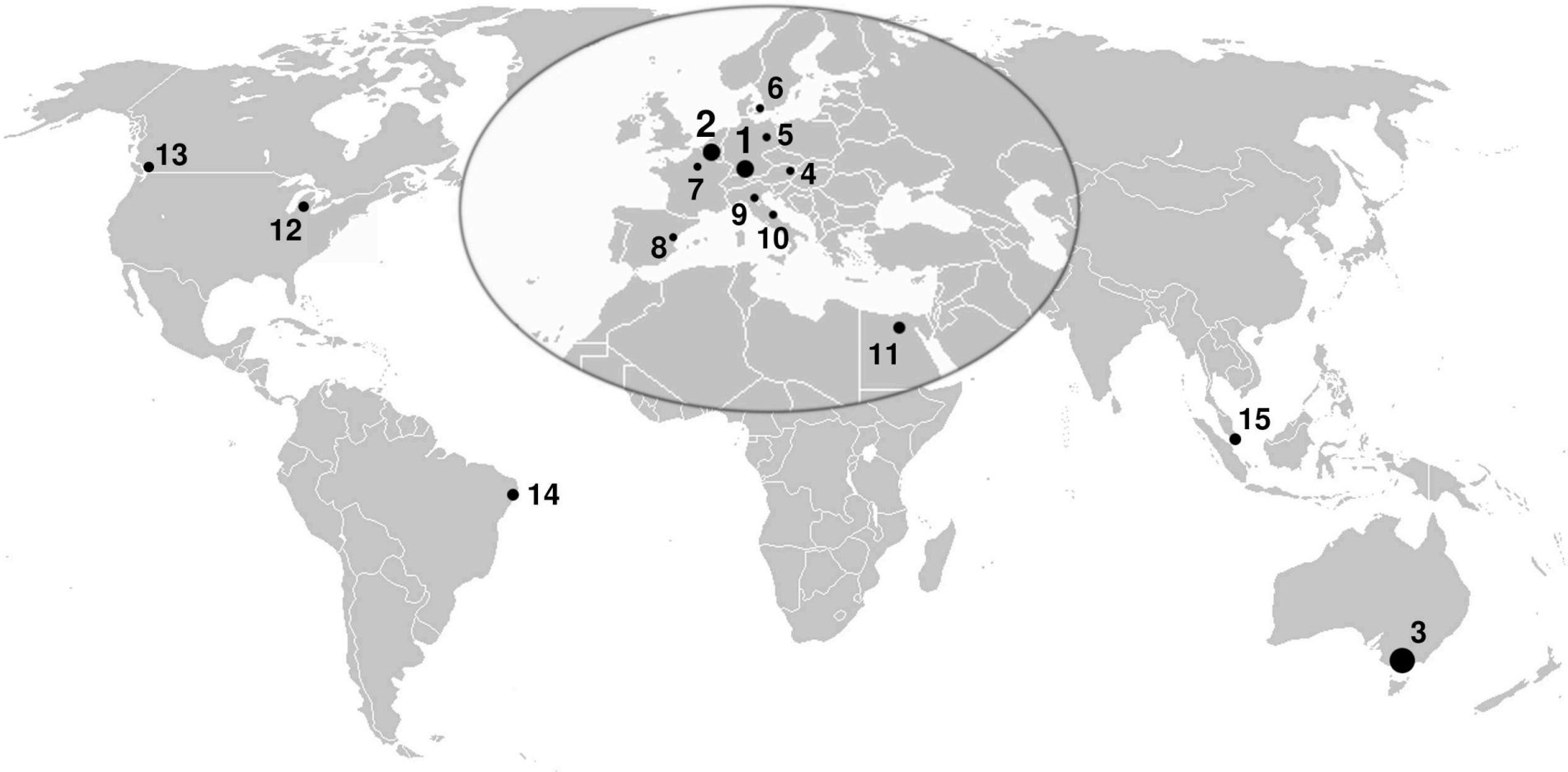
- completion (Frühwirth&Abdennadher)
- termination (Frühwirth;Voets&Pilozzi)
- complexity (Frühwirth;De Koninck)

Applications

- type systems (Sulzmann & Stuckey)
- test case generation (Schrijvers)
- multi-agent systems (Alberti)
- ...

CHR Researchers around the World



CHR in Industry

- multi-headed business rules
- custom constraint solvers

CHR in Industry

- multi-headed business rules
- custom constraint solvers

- Scientific Software & Systems Ltd.
  - stock brokering software

- Cornerstone Technology Inc
  - injection mould design tool

- BSSE System and Software Engineering
  - test generation

- MITRE Corporation
  - optical network design

Further Reading

- CHR website (Google for)
  - programs
  - papers
- CHR Survey
  - As Time Goes By: Constraint Handling Rules – A Survey of CHR Research from 1998 to 2007, J. Sneyers, P. Van Weert, T. Schrijvers, L. De Koninck.

Thank You!

My Questions: Get the Quiz!

Your Questions?