

# Cool War

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In year 6002 planet Earth has reached a high degree of civilization and war has essentially disappeared. Only a vestige of war survives in the way international controversies between countries are resolved: The two opponent countries, say C1 and C2, set up a square battlefield and they turn in placing their tanks in the field. Tanks cannot overlap. The winner is the last country which is able to place a tank.

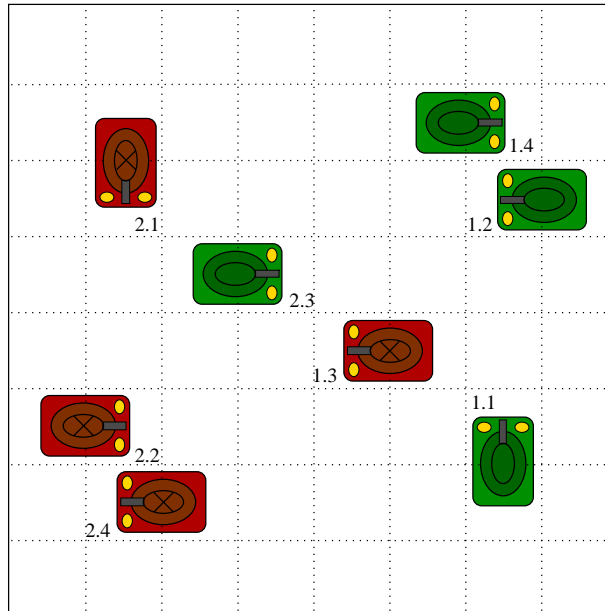
To make the process more disciplined, the battlefield is divided into  $n \times n$  squares (as a kind of chessboard), where  $n$  is even, and each tank occupies two adjacent squares, horizontally in a row or vertically in a column. As an example, you can see below a possible configuration of the field after four turns. The tanks placed by C1 are green, while those of C2 are red (and they bring a cross on the top, in case you have a black and white printout).



The battle is normally very quiet . . . the two opponent countries are quite nervous only when they have to decide who should start placing the first tank.

**Q1.** *Can you explain why? If C1 is the country that starts placing the tanks, does a winning strategy for C1 or for C2 exist?*

**Answer:** Yes, there is a winning strategy for C2. If C1 places a tank at a certain position, say  $p$ , C2 always answer by placing a tank at a position  $p'$  which is symmetric to  $p$  with respect to the center of the field. Since this maintains the symmetry of the field, whenever C1 is able to place a tank, also C2 will be able to do so, and thus C2 will surely be the winner. A possible configuration of the field after four steps is exemplified below. Tanks are numbered as  $x.y$  where  $x$  stands for the owner and  $y$  is a progressive number denoting at which turn the tank has been placed.



Note that the above argument applies to the case of “ordinary” tanks, which need a free path to access their final position, and also to flying tanks, which can be placed in any free position in the field, even if it not accessible.

Once the existence of a winning strategy is discovered, the regulation is drastically changed: the battlefield is no more organised in a grid, and now the tanks can be placed in any position, with any orientation, but still avoiding any overlapping.

**Q2.** *Do you think that this improves the situation?*

**Answer:** Not really. Now there is a winning strategy for C1. In fact, C1 can start placing the tank at the center of the board. Then, the previously illustrated strategy can be reused: at any turn C1 will play symmetrically with respect to

the move of C2 and thus C1 will be the winner. A possible configuration of the field, after five moves of C1 and four moves of C2 can be found below.

