

Extra Exercises

1 Resolution in Predicate Logic

1.1 Houses

I have the following information about houses:

- All houses are blue or red or big. (*Not necessarily exclusive*)
- Country houses are not expensive.
- Expensive houses are big or not red.
- If a house is big then, if it is not mine then it is a country house.
- If all houses are country houses, then there exists a house that is blue and red.
- Houses are expensive.
- No house is mine.

Convert all this information into first order predicate logic. To do this, you should use the predicates **blue(x)**, **red(x)**, **big(x)**, **country(x)**, **expensive(x)**, **mine(x)**. We assume that all individuals in this information are houses, so that there is **no** need to use a relation **house(x)**.

Next, use automated reasoning based on resolution (including proof through inconsistency and normalisation) to prove that the following statement is a logical consequence: “All houses are blue”.

1.2 Hats

Given is the following information:

- Every hat is white or black. (*No need to make this an exclusive or*)
- Every man has a hat that belongs to him.
- Nothing belongs to both John and Mark.
- All black hats belong to John.

Convert all this information into first order predicate logic. Use the constants `John`, `Mark`, and use the predicates `hat(x)`, `white(x)`, `black(x)`, `man(x)` and `belongs_to(x,y)`.

Now, use automated reasoning based on resolution (including proof by inconsistency and normalisation) to prove the sentence: “If Mark is a man, then there exists a white hat”.

Hint: it is always a good idea to start from factual information (a single true atom) and try to combine this with other formulae to get more factual information.

1.3 Countries

On the continent of Ozs there are 3 countries: Frigor, Caldar and Modo. The following information is known concerning Ozs:

- Caldar is warm.
- Modo is a neighbour of Frigor.
- Caldar is not a neighbour of a warm country.
- Frigor and Modo have the same neighbours.
- Every country has a neighbour.
- Every country is either warm or cold.
- If a country is a neighbour of another country, than the latter country is also a neighbour of the former.

Express the information above in first order predicate logic. To do this, make use of the constants `Frigor`, `Caldar` and `Modo`, and the relations `neighbour(x,y)`, (*meaning: x is a neighbour of y*), `warm(x)` and `cold(x)`.

Because the knowledge only talks of objects which are countries, it is not necessary to make use of a predicate `country(x)`. We will implicitly assume that anything in the knowledge-base is a country, so that we do not have to state this anywhere.

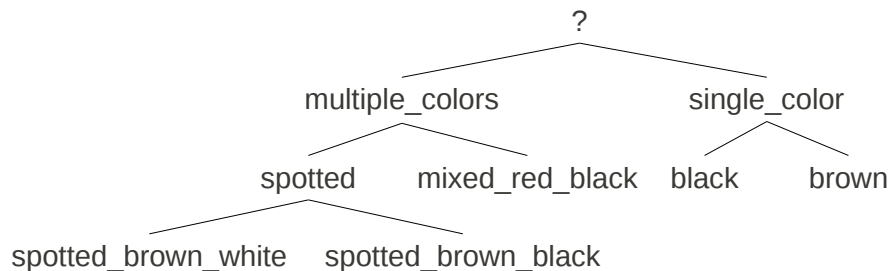
We now want to prove that there exists a cold country which is a neighbour of a warm country. Express this knowledge in a formula, using the above relations. Then use resolution based Automated reasoning (including normalisation and proof by inconsistency) to prove that the formulae follows from the knowledge.

2 Version Spaces

Marian wants to buy a dog. Unfortunately, she really has no preferences at all regarding which type of dog she wants to buy. However, she has many friends who have dogs. Therefore, she decides to ask her friends some questions about their dogs. In particular, she asks them about the size of their dog, about its social behavior, about whether it barks a lot and about its color. Finally, she also asks them whether they are happy with their dog. The answers of her friends are in the following table:

Size	Behaviour	Barks	Color	Happy?
large	friendly	a_lot	spotted_brown_white	yes
small	aggressive	little	black	no
large	aggressive	a_lot	spotted_brown_black	yes
medium	friendly	a_lot	mixed_red_black	no
small	friendly	a_lot	spotted_brown_black	yes

Marian then proceeds by applying the Version Space algorithm (learning by managing models) to determine which type of dog she should buy. To do this, she makes use of a language of 4-tuples of the form `[?,friendly,?,brown]`. Moreover, she also makes use of a hierarchy of color descriptions. These descriptions are represented in the following tree. Descriptions higher in the tree, connected by an arc to descriptions lower in the tree, are more general than the lower ones.



As a consequence, the description `[large,?,?,?]` is more general than `[large,?,?,multiple_colors]`, is more general than `[large,?,?,spotted]`, which is more general than `[large,?,?,spotted_brown_white]`.

Apply the version spaces algorithm to this problem. Show step by step how the hypothesis spaces G and S evolve. Indicate clearly whenever some hypothesis gets pruned and state why it is pruned. What concept(s) is(are) induced? Can Marian already classify the following extra examples on the basis of what she learned?

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[medium,friendly,little,spotted_brown_white]
[large,aggressive,a_lot,spotted_brown_black]
[small,aggressive,a_lot,mixed_red_black]
  
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