

Exercises: Artificial Intelligence

Automated Reasoning:
Movable Objects

Automated Reasoning: Movable Objects

PROBLEM

Problem: Movable Objects

- ***Given:***

- If all movable objects are blue, then all non-movable objects are green.
- If there exists a non-movable object, then all movable objects are blue.
- D is a non-movable object.

- ***Prove by resolution:***

- There exists a green object.

Automated Reasoning: Resolution

SOLUTION

Solution: Movable Objects

- ***English to logic***
- ***Logic to implicative normal form***
 - *Model*
 - *Assumption to prove*
- ***Apply resolution***
 - *Derive inconsistency:*
 - *Model + negated assumption*

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
- If there exists a non-movable object, then all movable objects are blue.
- D is a non-movable object.

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
 - $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
- If there exists a non-movable object, then all movable objects are blue.
- D is a non-movable object.

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
 - $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
- If there exists a non-movable object, then all movable objects are blue.
 - $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
- D is a non-movable object.

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
 - $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
- If there exists a non-movable object, then all movable objects are blue.
 - $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
- D is a non-movable object.
 - $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
– $\neg(\forall x \neg \text{mov}(x) \vee \text{blue}(x)) \vee (\forall y \text{ mov}(y) \vee \text{green}(y))$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\neg(\forall x \neg \text{mov}(x) \vee \text{blue}(x)) \vee (\forall y \text{ mov}(y) \vee \text{green}(y))$
 - $\forall y (\exists x \text{ mov}(x) \wedge \neg \text{blue}(x)) \vee \text{mov}(y) \vee \text{green}(y)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- **$(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$**
 - $\neg(\forall x \neg \text{mov}(x) \vee \text{blue}(x)) \vee (\forall y \text{ mov}(y) \vee \text{green}(y))$
 - $\forall y (\exists x \text{ mov}(x) \wedge \neg \text{blue}(x)) \vee \text{mov}(y) \vee \text{green}(y)$
 - **$\forall y (\text{mov}(A) \wedge \neg \text{blue}(A)) \vee \text{mov}(y) \vee \text{green}(y)$**
- **$(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$**
- **$\neg \text{mov}(D)$**

Solution: Implicative normal form

- **$(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$**
 - $\neg(\forall x \neg \text{mov}(x) \vee \text{blue}(x)) \vee (\forall y \text{ mov}(y) \vee \text{green}(y))$
 - $\forall y (\exists x \text{ mov}(x) \wedge \neg \text{blue}(x)) \vee \text{mov}(y) \vee \text{green}(y)$
 - $\forall y (\text{mov}(A) \wedge \neg \text{blue}(A)) \vee \text{mov}(y) \vee \text{green}(y)$
 - **$\forall y (\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y)) \wedge (\neg \text{blue}(A) \vee \text{mov}(y) \vee \text{green}(y))$**
- **$(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$**
- **$\neg \text{mov}(D)$**

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) (\leftarrow \text{true})$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) (\leftarrow \text{true})$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
 - $\neg(\exists x \neg \text{mov}(x)) \vee (\forall y \neg \text{mov}(y) \vee \text{blue}(y))$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) (\leftarrow \text{true})$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
 - $\neg(\exists x \neg \text{mov}(x)) \vee (\forall y \neg \text{mov}(y) \vee \text{blue}(y))$
 - $\forall x \forall y \text{ mov}(x) \vee \neg \text{mov}(y) \vee \text{blue}(y)$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) \leftarrow \text{true}$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
 - $\neg(\exists x \neg \text{mov}(x)) \vee (\forall y \neg \text{mov}(y) \vee \text{blue}(y))$
 - $\forall x \forall y \text{ mov}(x) \vee \neg \text{mov}(y) \vee \text{blue}(y)$
 - $\forall x \forall y \text{ mov}(y) \rightarrow \text{mov}(x) \vee \text{blue}(y)$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) \leftarrow \text{true}$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
 - $\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$
- $\neg \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) \leftarrow \text{true}$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
 - $\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$
- $\neg \text{mov}(D)$
 - $\text{false} \leftarrow \text{mov}(D)$

Solution: Implicative normal form

- $(\forall x \text{ mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y \neg \text{mov}(y) \rightarrow \text{green}(y))$
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) \leftarrow \text{true}$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
- $(\exists x \neg \text{mov}(x)) \rightarrow (\forall y \text{ mov}(y) \rightarrow \text{blue}(y))$
 - $\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$
- $\neg \text{mov}(D)$
 - $\text{false} \leftarrow \text{mov}(D)$
- Negated assumption: $\neg \exists x \text{ green}(x) \leftrightarrow \forall x \neg \text{green}(x)$
 - $\text{false} \leftarrow \text{green}(x)$

Solution: Implicative normal form

- **Prove** using resolution:
 - Assumption: **false** \leftarrow **green(x)**
- **Model:**
 - $\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y) (\leftarrow \text{true})$
 - $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$
 - $\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$
 - $\text{false} \leftarrow \text{mov}(D)$

Solution: Resolution

$\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$

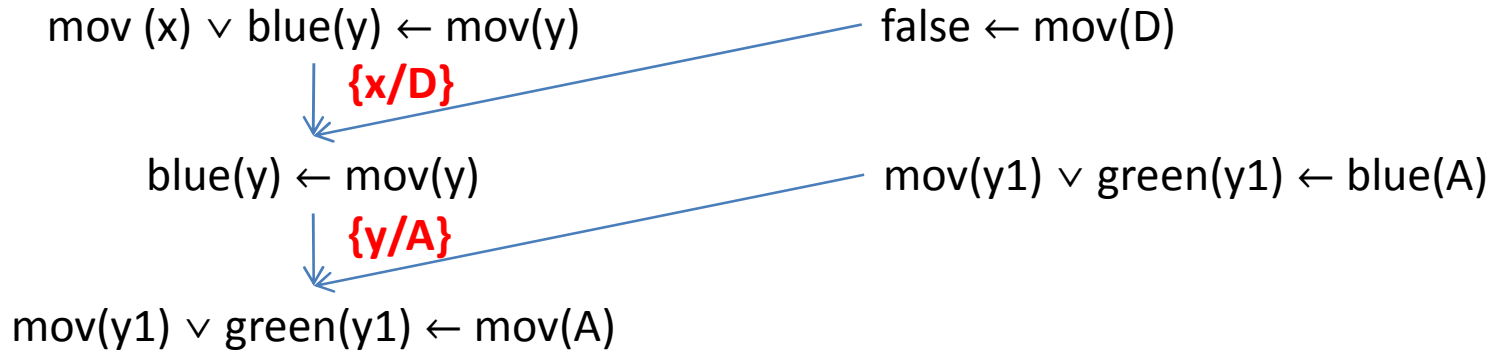
Solution: Resolution

$\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$ $\text{false} \leftarrow \text{mov}(D)$

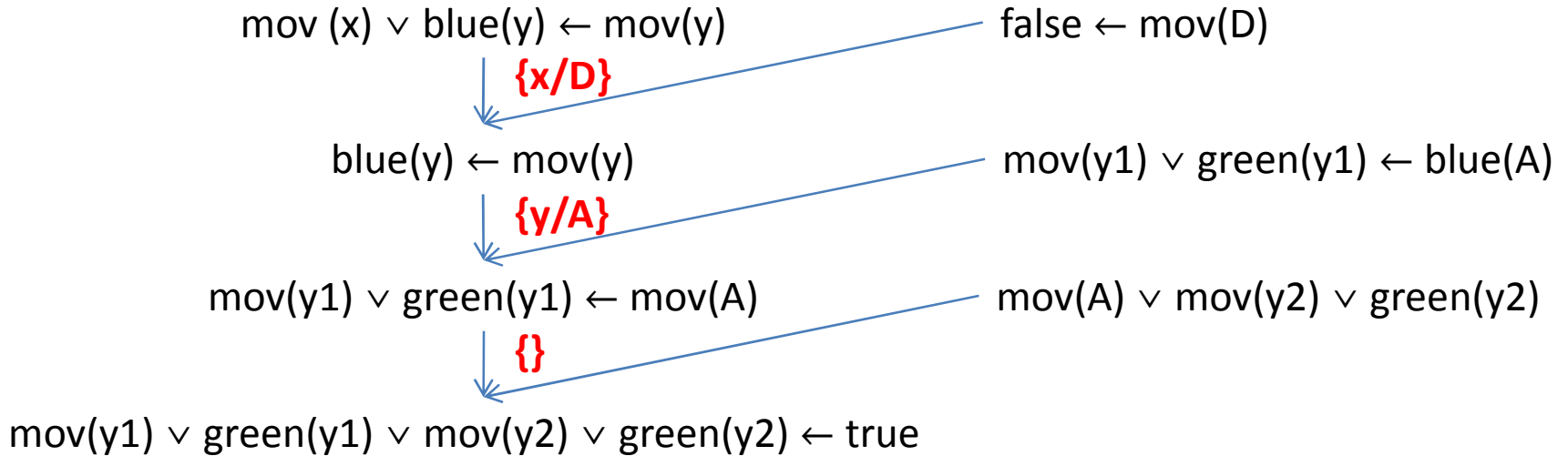
\downarrow **{x/D}**

$\text{blue}(y) \leftarrow \text{mov}(y)$

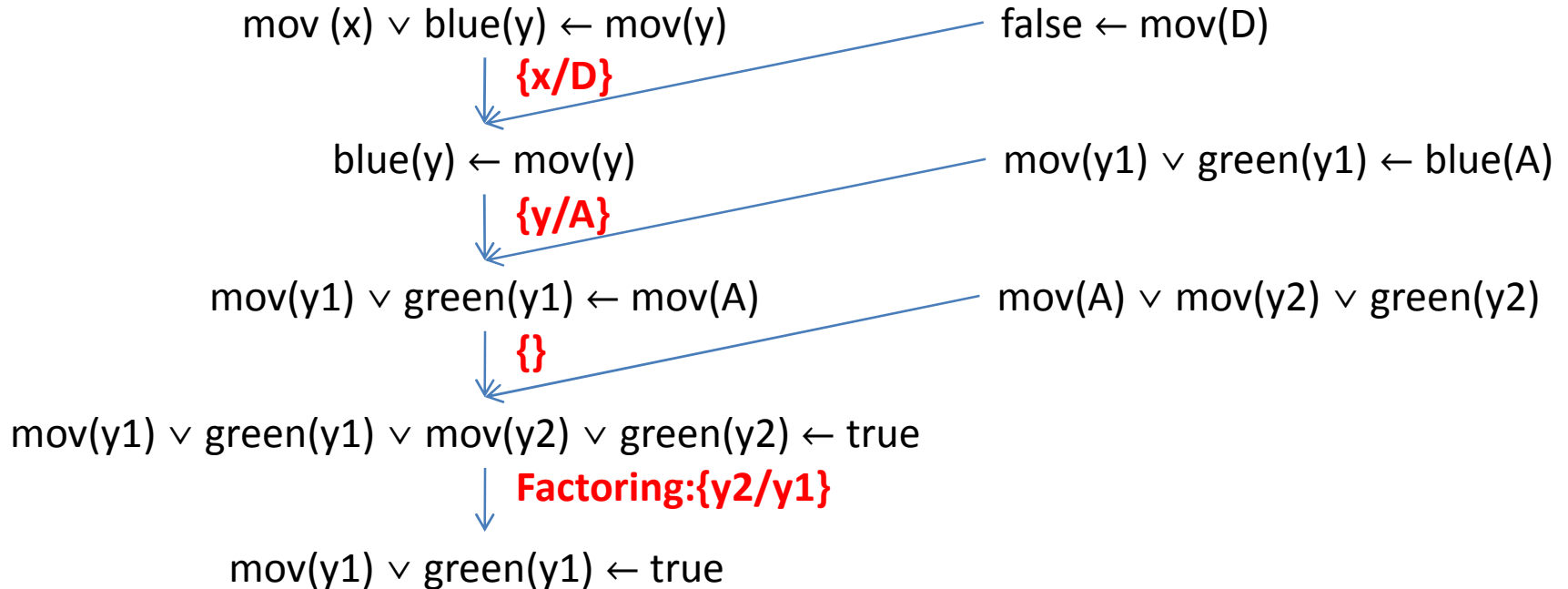
Solution: Resolution



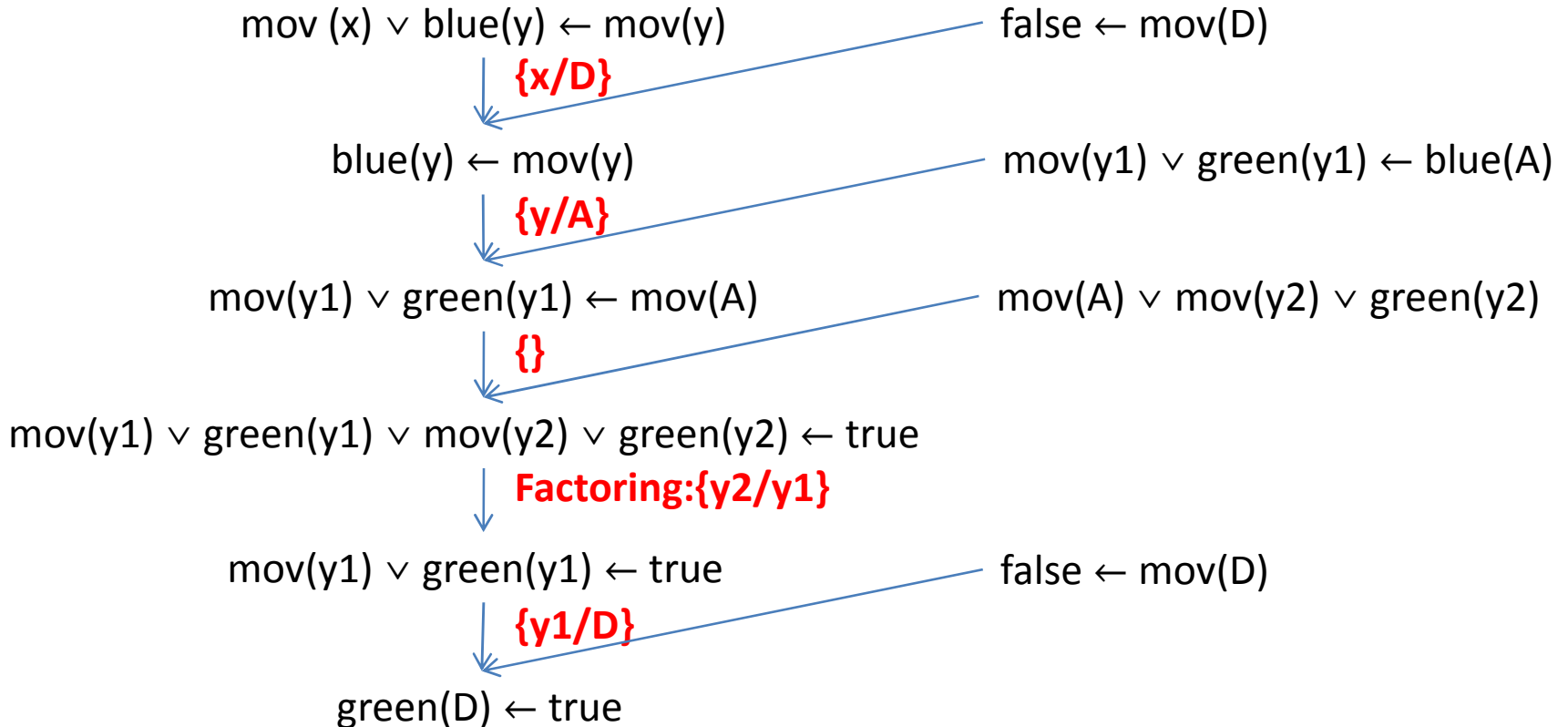
Solution: Resolution



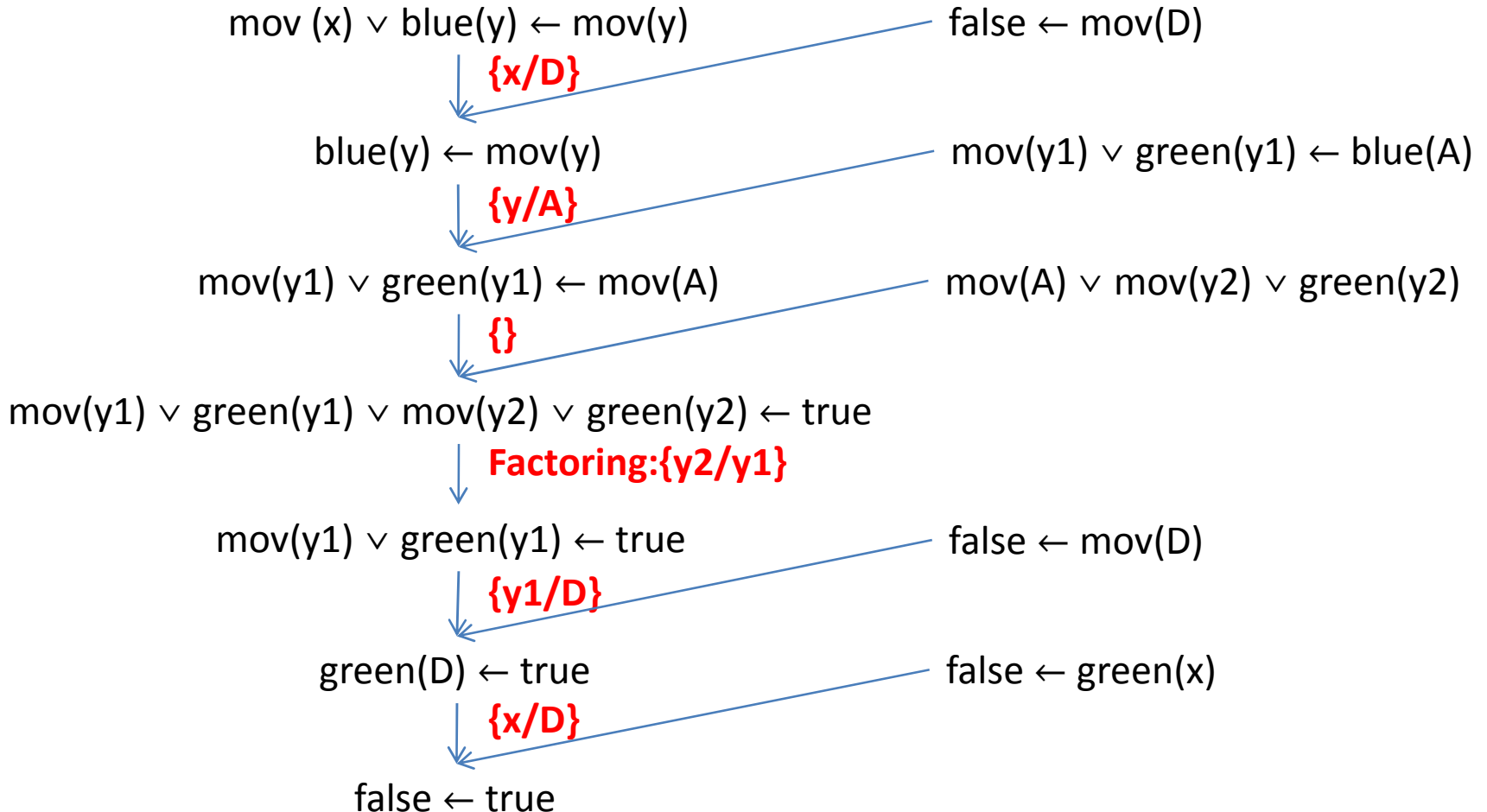
Solution: Resolution



Solution: Resolution



Solution: Resolution



Exercises: Artificial Intelligence

Automated Reasoning: Politicians

Automated Reasoning: Politicians

PROBLEM

Problem: Politicians

- ***Given:***

- If a poor politician exists, then all politicians are male.
- If people are friends with a politician, then this politician is poor and female.
- Lazy people have no friends.
- People are either male or female, but not both.
- If Joel is not lazy, then he is a politician.

- ***Proof by resolution:***

- There exists no person who is a friend of Joel.

Automated Reasoning: Politicians

SOLUTION

Solution: Politicians

- ***English to logic***
- ***Logic to implicative normal form***
 - *Model*
 - *Assumption to prove*
- ***Resolution***
 - *Derive inconsistency:*
 - *model + negated assumption*

Automated Reasoning: Politicians

SOLUTION: ENGLISH TO LOGIC

Solution: English to logic

- If a poor politician exists, then all politicians are male.
 - $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y))$.
- If people are friends with a politician, then this politician is poor and female.
- Lazy people have no friends.
- People are either male or female, but not both.
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y)).$
- If people are friends with a politician, then this politician is poor and female.
 - $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x).$
- Lazy people have no friends.
- People are either male or female, but not both.
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y)).$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x).$
- Lazy people have no friends.
 - $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x))).$
- People are either male or female, but not both.
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y)).$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x).$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x))).$
- People are either male or female, but not both.
 - $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x))).$
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y)).$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x).$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x))).$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x))).$
- If Joel is not lazy, then he is a politician.
 - $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel}).$

Solution: English to logic

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y)).$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x).$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x))).$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x))).$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel}).$

Automated Reasoning: Politicians

SOLUTION: IMPLICATIVE NORMAL FORM

Solution: Implicative normal form

- **$(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y))$**
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y))$
– $\neg \neg(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \vee (\forall y \neg \text{pol}(y) \vee \text{male}(y))$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y))$
 - $\neg(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \vee (\forall y \neg \text{pol}(y) \vee \text{male}(y))$
 - **$\forall x \forall y \neg \text{pol}(x) \vee \neg \text{poor}(x) \vee \neg \text{pol}(y) \vee \text{male}(y)$**
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y))$
 - $\neg(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \vee (\forall y \neg \text{pol}(y) \vee \text{male}(y))$
 - $\forall x \forall y \neg \text{pol}(x) \vee \neg \text{poor}(x) \vee \neg \text{pol}(y) \vee \text{male}(y)$
 - **$\forall x \forall y \neg(\text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)) \vee \text{male}(y)$**
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \rightarrow (\forall y \text{ pol}(y) \rightarrow \text{male}(y))$
 - $\neg(\exists x \text{ pol}(x) \wedge \text{poor}(x)) \vee (\forall y \neg \text{pol}(y) \vee \text{male}(y))$
 - $\forall x \forall y \neg \text{pol}(x) \vee \neg \text{poor}(x) \vee \neg \text{pol}(y) \vee \text{male}(y)$
 - $\forall x \forall y \neg(\text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)) \vee \text{male}(y)$
 - **$\forall x \forall y \text{ pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y) \rightarrow \text{male}(y)$**
- $\forall x (\text{pol}(x) \wedge (\exists y \text{ fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
- $\forall x \text{lazy}(x) \rightarrow (\neg(\exists y \text{fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
– $\forall x \neg(\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \vee (\text{poor}(x) \wedge \text{fem}(x))$
- $\forall x \text{lazy}(x) \rightarrow (\neg(\exists y \text{fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
 - $\forall x \neg(\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \neg\text{pol}(x) \vee \neg(\exists y \text{fr}(y,x)) \vee (\text{poor}(x) \wedge \text{fem}(x))$
- $\forall x \text{lazy}(x) \rightarrow (\neg(\exists y \text{fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg\text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
 - $\forall x \neg(\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \neg\text{pol}(x) \vee \neg(\exists y \text{fr}(y,x)) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - **$\forall x \forall y \neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee (\text{poor}(x) \wedge \text{fem}(x))$**
- $\forall x \text{lazy}(x) \rightarrow (\neg(\exists y \text{fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg\text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
 - $\forall x \neg(\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \neg\text{pol}(x) \vee \neg(\exists y \text{fr}(y,x)) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \forall y \neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \forall y (\neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee \text{poor}(x)) \wedge (\neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee \text{fem}(x))$
- $\forall x \text{lazy}(x) \rightarrow (\neg(\exists y \text{fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg\text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
 - $\forall x \neg(\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \neg\text{pol}(x) \vee \neg(\exists y \text{fr}(y,x)) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \forall y \neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \forall y (\neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee \text{poor}(x)) \wedge (\neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee \text{fem}(x))$
 - $\forall x \forall y (\neg(\text{pol}(x) \wedge \text{fr}(y,x)) \vee \text{poor}(x)) \wedge (\neg(\text{pol}(x) \wedge \text{fr}(y,x)) \vee \text{fem}(x))$
- $\forall x \text{lazy}(x) \rightarrow (\neg(\exists y \text{fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg\text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\forall x (\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \rightarrow \text{poor}(x) \wedge \text{fem}(x)$
 - $\forall x \neg(\text{pol}(x) \wedge (\exists y \text{fr}(y,x))) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \neg\text{pol}(x) \vee \neg(\exists y \text{fr}(y,x)) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \forall y \neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee (\text{poor}(x) \wedge \text{fem}(x))$
 - $\forall x \forall y (\neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee \text{poor}(x)) \wedge (\neg\text{pol}(x) \vee \neg\text{fr}(y,x) \vee \text{fem}(x))$
 - $\forall x \forall y (\neg(\text{pol}(x) \wedge \text{fr}(y,x)) \vee \text{poor}(x)) \wedge (\neg(\text{pol}(x) \wedge \text{fr}(y,x)) \vee \text{fem}(x))$
 - $\forall x \forall y (\text{pol}(x) \wedge \text{fr}(y,x) \rightarrow \text{poor}(x)) \wedge (\text{pol}(x) \wedge \text{fr}(y,x) \rightarrow \text{fem}(x))$
- $\forall x \text{lazy}(x) \rightarrow (\neg(\exists y \text{fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg\text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- **$\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$**
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
 - $\forall x \neg \text{lazy}(x) \vee (\neg(\exists y \text{ fr}(y,x)))$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
 - $\forall x \neg \text{lazy}(x) \vee (\neg(\exists y \text{ fr}(y,x)))$
 - **$\forall x \neg \text{lazy}(x) \vee \forall y \neg \text{fr}(y,x)$**
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
 - $\forall x \neg \text{lazy}(x) \vee (\neg(\exists y \text{ fr}(y,x)))$
 - $\forall x \neg \text{lazy}(x) \vee \forall y \neg \text{fr}(y,x)$
 - **$\forall x \forall y \neg(\text{lazy}(x) \wedge \text{fr}(y,x)) \vee \text{false}$**
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\forall x \text{ lazy}(x) \rightarrow (\neg(\exists y \text{ fr}(y,x)))$
 - $\forall x \neg \text{lazy}(x) \vee (\neg(\exists y \text{ fr}(y,x)))$
 - $\forall x \neg \text{lazy}(x) \vee \forall y \neg \text{fr}(y,x)$
 - $\forall x \forall y \neg(\text{lazy}(x) \wedge \text{fr}(y,x)) \vee \text{false}$
 - $\forall x \forall y \text{ lazy}(x) \wedge \text{fr}(y,x) \rightarrow \text{false}$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{false} \leftarrow \text{lazy}(x) \wedge \text{fr}(y,x)$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{false} \leftarrow \text{lazy}(x) \wedge \text{fr}(y,x)$
- $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\neg(\text{male}(x) \wedge \text{fem}(x)))$
 - $\forall x (\text{male}(x) \vee \text{fem}(x)) \wedge (\text{male}(x) \wedge \text{fem}(x) \rightarrow \text{false})$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{false} \leftarrow \text{lazy}(x) \wedge \text{fr}(y,x)$
- $\text{male}(x) \vee \text{fem}(x)$
- $\text{false} \leftarrow \text{male}(x) \wedge \text{fem}(x)$
- **$\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$**

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{false} \leftarrow \text{lazy}(x) \wedge \text{fr}(y,x)$
- $\text{male}(x) \vee \text{fem}(x)$
- $\text{false} \leftarrow \text{male}(x) \wedge \text{fem}(x)$
- $\neg \text{lazy}(\text{Joel}) \rightarrow \text{pol}(\text{Joel})$
 - **$\text{lazy}(\text{Joel}) \vee \text{pol}(\text{Joel})$**

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{false} \leftarrow \text{lazy}(x) \wedge \text{fr}(y,x)$
- $\text{male}(x) \vee \text{fem}(x)$
- $\text{false} \leftarrow \text{male}(x) \wedge \text{fem}(x)$
- $\text{lazy}(\text{Joel}) \vee \text{pol}(\text{Joel})$

Solution: Implicative normal form

- Prove:
 - There exists no person who is a friend of Joel
 - $\neg \exists x \text{ fr}(x, \text{Joel}) \leftrightarrow \forall x \neg \text{fr}(x, \text{Joel})$
- Negate assumption:
 - There exists a person who is a friend of Joel
 - $\exists x \text{ fr}(x, \text{Joel})$
 - Call the friend S
 - $\text{fr}(S, \text{Joel})$

Solution: Implicative normal form

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{false} \leftarrow \text{lazy}(x) \wedge \text{fr}(y,x)$
- $\text{male}(x) \vee \text{fem}(x)$
- $\text{false} \leftarrow \text{male}(x) \wedge \text{fem}(x)$
- $\text{lazy}(\text{Joel}) \vee \text{pol}(\text{Joel})$
- **$\text{fr}(\text{S},\text{Joel})$**

Automated Reasoning: Politicians

SOLUTION: RESOLUTION

Solution: Apply Resolution

- $\text{male}(y) \leftarrow \text{pol}(x) \wedge \text{poor}(x) \wedge \text{pol}(y)$
- $\text{poor}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{fem}(x) \leftarrow \text{pol}(x) \wedge \text{fr}(y,x)$
- $\text{false} \leftarrow \text{lazy}(x) \wedge \text{fr}(y,x)$
- $\text{male}(x) \vee \text{fem}(x)$
- $\text{false} \leftarrow \text{male}(x) \wedge \text{fem}(x)$
- $\text{lazy}(\text{Joel}) \vee \text{pol}(\text{Joel})$
- $\text{fr}(\text{S}, \text{Joel})$

Solution: Apply Resolution

- **male(y1) ← pol(x1) ∧ poor(x1) ∧ pol(y1)**
 - poor(x2) ← pol(x2) ∧ fr(y2,x2)

Solution: Apply Resolution

- $\text{male}(y1) \leftarrow \text{pol}(x1) \wedge \underline{\text{poor}(x1)} \wedge \text{pol}(y1)$
 - $\underline{\text{poor}(x2)} \leftarrow \text{pol}(x2) \wedge \text{fr}(y2,x2)$
 - RESOLUTION: $\{x2/x1\}$
- $\text{male}(y1) \leftarrow \text{pol}(x1) \wedge \text{pol}(y1) \wedge \text{fr}(y2,x1)$

Solution: Apply Resolution

- $\text{male}(y1) \leftarrow \text{pol}(x1) \wedge \underline{\text{poor}(x1)} \wedge \text{pol}(y1)$
 - $\underline{\text{poor}(x2)} \leftarrow \text{pol}(x2) \wedge \text{fr}(y2,x2)$
 - RESOLUTION: $\{x2/x1\}$
- $\text{male}(y1) \leftarrow \underline{\text{pol}(x1)} \wedge \underline{\text{pol}(y1)} \wedge \text{fr}(y2,x1)$

Solution: Apply Resolution

- $\text{male}(y1) \leftarrow \text{pol}(x1) \wedge \underline{\text{poor}(x1)} \wedge \text{pol}(y1)$
 - $\underline{\text{poor}(x2)} \leftarrow \text{pol}(x2) \wedge \text{fr}(y2,x2)$
 - RESOLUTION: $\{x2/x1\}$
- $\text{male}(y1) \leftarrow \underline{\text{pol}(x1)} \wedge \underline{\text{pol}(y1)} \wedge \text{fr}(y2,x1)$
 - FACTORING: $\{y1/x1\}$
- $\text{male}(x1) \leftarrow \text{pol}(x1) \wedge \text{fr}(y2,x1)$
 - *'Politicians who have friends must be male'*

Solution: Apply Resolution

- male(x1) \leftarrow pol(x1) \wedge fr(y2,x1)
 - false \leftarrow male(x3) \wedge fem(x3)

Solution: Apply Resolution

- **male(x1) \leftarrow pol(x1) \wedge fr(y2,x1)**
 - false \leftarrow male(x3) \wedge fem(x3)
 - RESOLUTION: {x3/x1}
- **false \leftarrow pol(x1) \wedge fr(y2,x1) \wedge fem(x1)**
 - *'Politicians who have friends cannot be female'*

Solution: Apply Resolution

- **false** \leftarrow **pol(x1) \wedge fr(y2,x1) \wedge fem(x1)**
 - fem(x4) \leftarrow pol(x4) \wedge fr(y4,x4)

Solution: Apply Resolution

- **false** \leftarrow **pol(x1) \wedge fr(y2,x1) \wedge fem(x1)**
 - fem(x4) \leftarrow pol(x4) \wedge fr(y4,x4)
 - RESOLUTION: {x4/x1}
- **false** \leftarrow **pol(x1) \wedge fr(y2,x1) \wedge fr(y4,x1)**

Solution: Apply Resolution

- **false** \leftarrow **pol(x1)** \wedge **fr(y2,x1)** \wedge **fem(x1)**
 - fem(x4) \leftarrow pol(x4) \wedge fr(y4,x4)
 - RESOLUTION: {x4/x1}
- **false** \leftarrow **pol(x1)** \wedge **fr(y2,x1)** \wedge **fr(y4,x1)**

Solution: Apply Resolution

- **false** \leftarrow **pol(x1) \wedge fr(y2,x1) \wedge fem(x1)**
 - fem(x4) \leftarrow pol(x4) \wedge fr(y4,x4)
 - RESOLUTION: {x4/x1}
- **false** \leftarrow **pol(x1) \wedge fr(y2,x1) \wedge fr(y4,x1)**
 - FACTORING: {y4/y2}
- **false** \leftarrow **pol(x1) \wedge fr(y2,x1)**
 - *'Politicians do not have friends'*

Solution: Apply Resolution

- **false** \leftarrow pol(x1) \wedge fr(y2,x1)
 - lazy(Joel) \vee pol(Joel)

Solution: Apply Resolution

- **false** \leftarrow **pol(x1)** \wedge **fr(y2,x1)**
 - lazy(Joel) \vee pol(Joel)
 - RESOLUTION: {x1/Joel}
- **lazy(Joel)** \leftarrow **fr(y2,Joel)**
 - *'If Joel has friend, then he must be lazy'*

Solution: Apply Resolution

- **lazy(Joel) \leftarrow fr(y2,Joel)**
 - false \leftarrow lazy(x5) \wedge fr(y5,x5)

Solution: Apply Resolution

- **lazy(Joel) \leftarrow fr(y2,Joel)**
 - false \leftarrow lazy(x5) \wedge fr(y5,x5)
 - RESOLUTION: {x5/Joel}
- **false \leftarrow fr(y2,Joel) \wedge fr(y5,Joel)**

Solution: Apply Resolution

- **lazy(Joel) \leftarrow fr(y2,Joel)**
 - false \leftarrow lazy(x5) \wedge fr(y5,x5)
 - RESOLUTION: {x5/Joel}
- false \leftarrow **fr(y2,Joel) \wedge fr(y5,Joel)**

Solution: Apply Resolution

- **lazy(Joel) \leftarrow fr(y2,Joel)**
 - false \leftarrow lazy(x5) \wedge fr(y5,x5)
 - RESOLUTION: {x5/Joel}
- false \leftarrow **fr(y2,Joel) \wedge fr(y5,Joel)**
 - FACTORING: {y5/y2}
- false \leftarrow fr(y2,Joel)
 - *'Joel does not have any friends'*

Solution: Apply Resolution

- **false** \leftarrow **fr(y2,Joel)**
– fr(S,Joel)

Solution: Apply Resolution

- **false** \leftarrow **fr(y2,Joel)**
 - fr(S,Joel)
 - RESOLUTION: {y2/S}
- **false** \leftarrow **true**