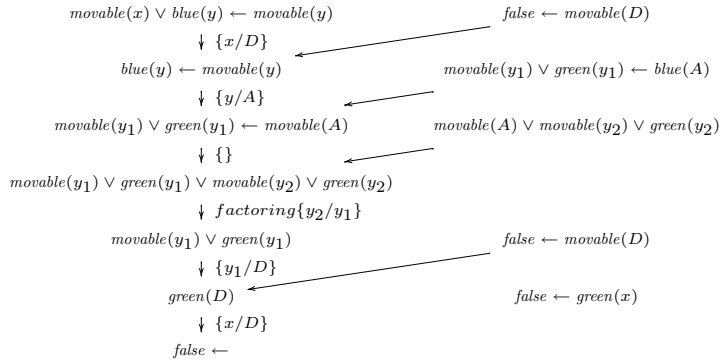


Movable Objects (3)



3

Movable Objects (1)

- If all movable objects are blue, then all non-movable objects are green.

$$\begin{aligned}
 & (\forall x \text{ movable}(x) \Rightarrow \text{blue}(x)) \Rightarrow (\forall y \neg \text{movable}(y) \Rightarrow \text{green}(y)) \\
 & \neg(\forall x \neg \text{movable}(x) \vee \text{blue}(x)) \vee \forall y \text{ movable}(y) \vee \text{green}(y) \\
 & \exists x (\text{movable}(x) \wedge \neg \text{blue}(x)) \vee \forall y \text{ movable}(y) \vee \text{green}(y) \\
 & \forall y (\text{movable}(A) \wedge \neg \text{blue}(A)) \vee \text{movable}(y) \vee \text{green}(y) \\
 & \forall y (\text{movable}(A) \vee \text{movable}(y) \vee \text{green}(y)) \wedge (\neg \text{blue}(A) \vee \text{movable}(y) \vee \text{green}(y)) \\
 & \left\{ \begin{array}{l} \text{movable}(A) \vee \text{movable}(y) \vee \text{green}(y) \\ \text{movable}(y) \vee \text{green}(y) \leftarrow \text{blue}(A) \end{array} \right\}
 \end{aligned}$$

1

Movable Objects (2)

- If there exists a non-movable object, then all movable objects are blue.

$$\begin{aligned}
 & (\exists x \neg \text{movable}(x)) \Rightarrow (\forall y \text{ movable}(y) \Rightarrow \text{blue}(y)) \\
 & \neg(\exists x \neg \text{movable}(x) \vee (\forall y \neg \text{movable}(y) \vee \text{blue}(y))) \\
 & \forall x \forall y \text{ movable}(x) \vee \neg \text{movable}(y) \vee \text{blue}(y) \\
 & \text{movable}(x) \vee \text{blue}(y) \leftarrow \text{movable}(y)
 \end{aligned}$$

- D is a non-movable object.

$$\begin{aligned}
 & \neg \text{movable}(D) \\
 & \text{false} \leftarrow \text{movable}(D)
 \end{aligned}$$

- There doesn't exist a green object.

$$\begin{aligned}
 & \forall x \neg \text{green}(x) \\
 & \text{false} \leftarrow \text{green}(x)
 \end{aligned}$$

2

- If there exists a **poor politician**, then all politicians are male.

$$\begin{aligned}
 & (\exists x p(x) \wedge \text{poor}(x)) \rightarrow (\forall y p(y) \rightarrow m(y)). \\
 & \neg(\exists x p(x) \wedge \text{poor}(x)) \vee (\forall y \neg p(y) \vee m(y)). \\
 & \forall x (\neg p(x) \vee \neg \text{poor}(x)) \vee (\forall y \neg p(y) \vee m(y)). \\
 & \rightsquigarrow m(y) \leftarrow p(x) \wedge \text{poor}(x) \wedge p(y).
 \end{aligned}$$

- If people are friends with a politician, then this politician is poor and female.

$$\begin{aligned}
 & \forall x p(x) \wedge (\exists y fr(y, x)) \rightarrow \text{poor}(x) \wedge f(x). \\
 & \forall x \neg(p(x) \wedge (\exists y fr(y, x)) \vee (\text{poor}(x) \wedge f(x))). \\
 & \forall x \neg p(x) \vee (\forall y \neg fr(y, x)) \vee (\text{poor}(x) \wedge f(x)). \\
 & \neg p(x) \vee \neg fr(y, x) \vee (\text{poor}(x) \wedge f(x)). \\
 & (\neg p(x) \vee \neg fr(y, x) \vee \text{poor}(x)) \\
 & \wedge (\neg p(x) \vee \neg fr(y, x) \vee f(x)). \\
 & \rightsquigarrow \left\{ \begin{array}{l} \text{poor}(x) \leftarrow p(x) \wedge fr(y, x). \\ f(x) \leftarrow p(x) \wedge fr(y, x). \end{array} \right\}
 \end{aligned}$$

4

Politics (2)

- Lazy people have no friends.

$$\begin{aligned} \forall x l(x) &\rightarrow (\neg \exists y fr(y, x)). \\ \forall x \neg l(x) \vee \forall y \neg fr(y, x). \\ \rightsquigarrow false &\leftarrow l(x) \wedge fr(y, x). \end{aligned}$$

- People are either male or female but not both.

$$\begin{aligned} \forall x (m(x) \vee f(x)) \wedge (\neg(m(x) \wedge f(x))). \\ \forall x (m(x) \vee f(x)) \wedge (\neg m(x) \vee \neg(f(x))). \\ \rightsquigarrow \left\{ \begin{array}{l} m(x) \vee f(x). \\ false \leftarrow m(x) \wedge f(x). \end{array} \right\} \end{aligned}$$

5

Politics (3)

- If Joel is not lazy then he is a politician.

$$\begin{aligned} \neg l(J) &\rightarrow p(J). \\ \rightsquigarrow l(J) \vee p(J). \end{aligned}$$

- There exists no person who is a friend of Joel
Negation: There exists a person who is a friend of Joel

$$fr(S, J).$$

6

Politics (4)

- $m(y) \leftarrow p(x) \wedge poor(x) \wedge p(y)$.
- $poor(x) \leftarrow p(x) \wedge fr(y, x)$.
- $f(x) \leftarrow p(x) \wedge fr(y, x)$.
- $false \leftarrow l(x) \wedge fr(y, x)$.
- $m(x) \vee f(x)$.
- $false \leftarrow m(x) \wedge f(x)$.
- $l(J) \vee p(J)$.
- $fr(S, J)$.

7

Politics (5)

$$\begin{array}{l} m(y_1) \leftarrow p(x_1) \wedge poor(x_1) \wedge p(y_1). \\ poor(x_2) \leftarrow p(x_2) \wedge fr(y_2, x_2). \\ \text{resolution: } \{x_2/x_1\} \end{array}$$

$$\begin{array}{l} m(y_1) \leftarrow p(x_1) \wedge p(y_1) \wedge fr(y_2, x_1). \\ \text{factoring: } \{y_1/x_1\} \\ m(x_1) \leftarrow p(x_1) \wedge fr(y_2, x_1). \end{array}$$

“Politicians who have friends must be male.”

$$\begin{array}{l} m(x_1) \leftarrow p(x_1) \wedge fr(y_2, x_1). \\ false \leftarrow m(x_3) \wedge f(x_3). \\ \text{resolution: } \{x_3/x_1\} \end{array}$$

$false \leftarrow p(x_1) \wedge fr(y_2, x_1) \wedge f(x_1)$.
“Politicians who have friends cannot be female.”

8

Politics (6)

$$\begin{array}{l} \text{false} \leftarrow p(x_1) \wedge fr(y_2, x_1) \wedge \overline{f(x_1)}. \\ \overline{f(x_4)} \leftarrow p(x_4) \wedge fr(y_4, x_4). \\ \hline \text{resolution: } \{x_4/x_1\} \end{array}$$

$$\begin{array}{l} \text{false} \leftarrow p(x_1) \wedge fr(y_2, x_1) \wedge fr(y_4, x_1). \\ \text{factoring: } \{y_4/y_2\} \\ \text{false} \leftarrow p(x_1) \wedge fr(y_2, x_1). \\ \text{"Politicians don't have friends."} \end{array}$$

$$\begin{array}{l} \text{false} \leftarrow \overline{p(x_1)} \wedge fr(y_2, x_1). \\ \overline{l(J)} \vee p(J). \\ \hline \text{resolution: } \{x_1/J\} \end{array}$$

$$\begin{array}{l} \overline{l(J)} \leftarrow fr(y_2, J). \\ \text{"If Joel has friends, he must be lazy."} \end{array}$$

Politics (7)

$$\begin{array}{l} \overline{l(J)} \leftarrow fr(y_2, J). \\ \text{false} \leftarrow \overline{l(x_5)} \wedge fr(y_5, x_5). \\ \hline \text{resolution: } \{x_5/J\} \end{array}$$

$$\begin{array}{l} \text{false} \leftarrow fr(y_2, J) \wedge fr(y_5, J). \\ \text{factoring: } \{y_5/y_2\} \\ \text{false} \leftarrow fr(y_2, J). \\ \text{"Joel does not have any friends."} \end{array}$$

$$\begin{array}{l} \text{false} \leftarrow \overline{fr(y_2, J)}. \\ \overline{fr(S, J)}. \\ \hline \text{resolution: } \{y_2/J\} \end{array}$$

■