Weather (1)

1. **It is raining, it is snowing or it is dry.**
   $raining \vee snowing \vee dry$

2. **It is warm.**
   $warm$

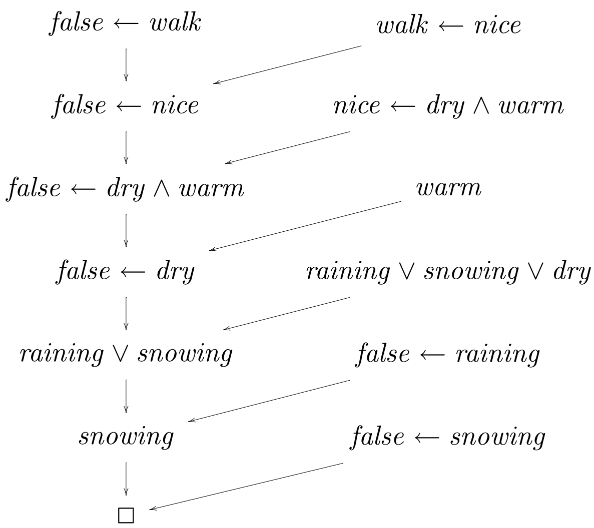
3. **It is not raining.**
   $false \leftarrow raining$ (or $\neg raining$)

4. **It is not snowing.**
   $false \leftarrow snowing$ (or $\neg snowing$)

5. **If the weather is nice, then it is good to walk.**
   $walk \leftarrow nice$

6. **If the weather is dry and warm, the weather is nice.**
   $nice \leftarrow dry \wedge warm$

Weather (2)



MGU (1)

Unify(a,b):
$mgu := \{a = b\}$;
$Stop :=$ false;
While not(Stop) and mgu still contains $s = t$ **of**
- **Case:** t is a variable, s is not a variable:
  $\rightarrow$ replace $s = t$ by $t = s$ in mgu;
- **Case:** s is a variable, t is the SAME variable:
  $\rightarrow$ delete $s = t$ from mgu;
- **Case:** s is a variable, t is not a variable and contains s:
  $\rightarrow$ Stop := true;
- **Case:** s is a variable, t is not identical to nor contains s and s occurse elsewher in mgu:
  $\rightarrow$ replace all other occurrences of s in mgu by t;
- **Case:** s is of the form $f(s_1, \ldots, s_n)$, t of $g(t_1, \ldots, t_m)$:
  $\rightarrow$ **if** $f \neq g$ or $n \neq m$:
  Stop := true;
  else
  replace $s = t$ in mgu by
  $s_1 = t_1,\ s_2 = t_2 \ldots s_n = t_n$

End While

MGU (2)

1. $p(f(y), w, g(z, y)) = p(x, x, g(z, A))$:

Init:	$p(f(y), w, g(z, y)) = p(x, x, g(z, A))$
Case 5:	$f(y) = x,\ w = x,\ g(z, y) = g(z, A)$
Case 1:	$x = f(y),\ w = x,\ g(z, y) = g(z, A)$
Case 4:	$x = f(y),\ w = f(y),\ g(z, y) = g(z, A)$
Case 5:	$x = f(y),\ w = f(y),\ z = z,\ y = A$
Case 2:	$x = f(y),\ w = f(y),\ y = A$
Case 4:	$x = f(A),\ w = f(A),\ y = A$

Mgu: $\{x/f(A),\ w/f(A),\ y/A\}$
Result: $p(f(A), f(A), g(z, A))$

2. $p(A, x, f(g(y))) = p(z, f(z), f(A))$

Init:	$p(A, x, f(g(y))) = p(z, f(z), f(A))$
Case 5:	$A = z,\ x = f(z),\ f(g(y)) = f(A)$
Case 1:	$z = A,\ x = f(z),\ f(g(y)) = f(A)$
Case 4:	$z = A,\ x = f(A),\ f(g(y)) = f(A)$
Case 5:	$z = A,\ x = f(A),\ g(y) = A$
Case 5:	`Stop:=true`

MGU (3)

1. $q(x,x) = q(y,f(y))$

   Init: $q(x,x) = q(y,f(y))$
   Case 5: $x = y$, $x = f(y)$
   Case 4: $x = y$, $y = f(y)$
   Case 3: Stop:=true

2. $f(x, g(f(a),u)) = f(g(u,v),x)$

   Init: $f(x,g(f(a),u)) = f(g(u,v),x)$
   Case 5: $x = g(u,v)$, $g(f(a),u) = x$
   Case 4: $x = g(u,v)$, $g(f(a),u) = g(u,v)$
   Case 5: $x = g(u,v)$, $f(a) = u$, $u = v$
   Case 1: $x = g(u,v)$, $u = f(a)$, $u = v$
   Case 4: $x = g(f(a),v)$, $u = f(a)$, $f(a) = v$
   Case 1: $x = g(f(a),v)$, $u = f(a)$, $v = f(a)$
   Case 4: $x = g(f(a),f(a))$, $u = f(a)$, $v = f(a)$

   Mgu: $\{x/g(f(a),f(a)),\ u/f(a),\ v/f(a)\}$
   Result: $f(g(f(a),f(a)),g(f(a),f(a)))$

Horn Clause Resolution (1)

$false \leftarrow b(n,b) \wedge b(b,g) \wedge b(g,n)$ $\qquad b(x_1,y_1) \leftarrow c(x_1) \wedge c(y_1) \wedge d(x_1,y_1)$

$\{x_1/n, y_1/b\}$

$false \leftarrow c(n) \wedge c(b) \wedge d(n,b) \wedge b(b,g) \wedge b(g,n)$ $\qquad b(x_2,y_2) \leftarrow c(x_2) \wedge c(y_2) \wedge d(x_2,y_2)$

$\{x_2/b, y_2/g\}$

$false \leftarrow c(n) \wedge c(b) \wedge d(n,b) \wedge c(g) \wedge d(b,g) \wedge b(g,n)$ $\qquad b(x_3,y_3) \leftarrow c(x_3) \wedge c(y_3) \wedge d(x_3,y_3)$

$\{x_3/b, y_3/g\}$

$false \leftarrow c(n) \wedge c(b) \wedge d(n,b) \wedge c(g) \wedge d(b,g) \wedge d(g,n)$

...

Horn Clause Resolution (2)

...

$false \leftarrow c(n) \wedge c(b) \wedge d(n,b) \wedge c(g) \wedge d(b,g) \wedge d(g,n)$ $\qquad c(R)$

$\{n/R\}$

$false \leftarrow c(b) \wedge d(R,b) \wedge c(g) \wedge d(b,g) \wedge d(g,R)$ $\qquad c(G)$

$\{b/G\}$

$false \leftarrow d(R,G) \wedge c(g) \wedge d(G,g) \wedge d(g,R)$ $\qquad c(B)$

$\{g/B\}$

$false \leftarrow d(R,G) \wedge d(G,B) \wedge d(B,R)$

...

Horn Clause Resolution (3)

...

$false \leftarrow d(R,G) \wedge d(G,B) \wedge d(B,R)$ $\qquad d(R,G)$

$false \leftarrow d(G,B) \wedge d(B,R)$ $\qquad d(G,B)$

$false \leftarrow d(B,R)$ $\qquad d(B,R)$

$\square$

Predicate Resolution (1)

- Add $\neg \forall x\ \exists y\ p(f(x)) \wedge r(y)$ to theory.

- Normalisation (implicative normal form)

  $p(x) \leftarrow r(f(x))$
  $r(f(x)) \vee r(f(f(y)))$
  $\exists x\ \forall y\ \neg [p(f(x)) \wedge r(y)] \leadsto \mathit{false} \leftarrow p(f(A)) \wedge r(y)$

- Resolution

$$\begin{array}{ccc}
false \leftarrow p(f(A)) \wedge r(y) & & p(x) \leftarrow r(f(x)) \\
\downarrow \{x/f(A)\} & \swarrow & \\
false \leftarrow r(f(f(A))) \wedge r(y) & & r(f(x_1)) \vee r(f(f(y_1))) \\
\text{factoring} \downarrow \{y/f(f(A))\} & & \text{factoring} \downarrow \{x_1/f(y_1)\} \\
false \leftarrow r(f(f(A))) & & r(f(f(y_1))) \\
\downarrow \{y_1/A\} & \swarrow & \\
false \leftarrow & &
\end{array}$$