

# Exercises: Artificial Intelligence

Constraint Processing II & Waltz: The  
4 Teachers problem

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# **PROBLEM**

# Problem

- Four teachers: A, B, C & D
  - A, B, C & D assigned rooms
  - 5 rooms available: 1, 2, 3, 4 & 5
  - $A \neq 1$
  - $B \neq 2$
  - room  $D \geq 3$
  - room  $D < \text{room } B$
  - $|C-B| \neq 1$
  - $C \neq 5$

# Problem

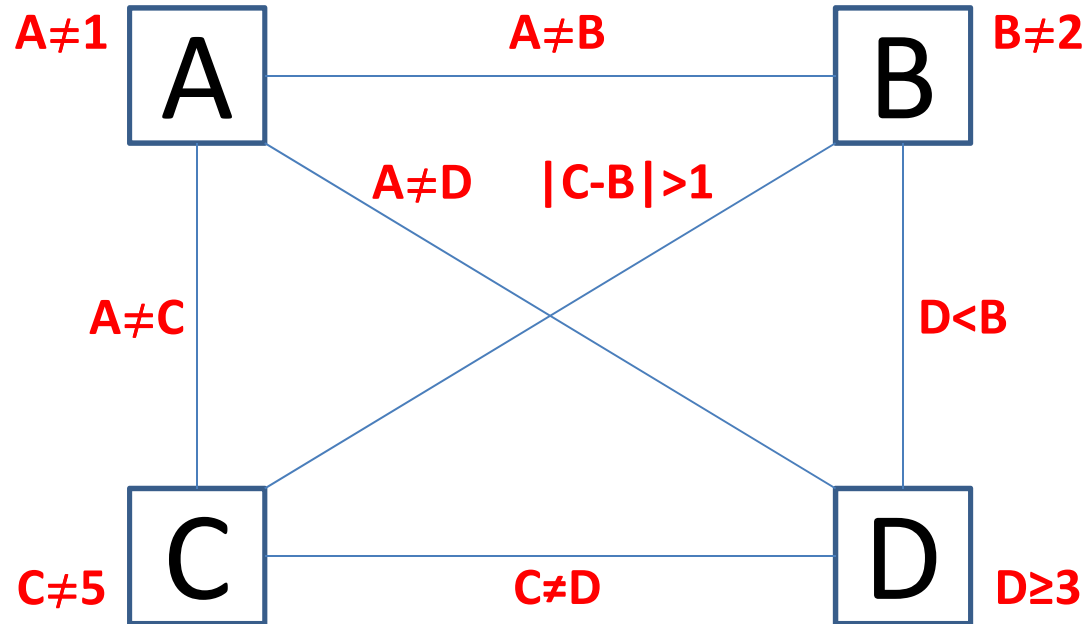
- Which family lives in which house?
- Solve with:
  - Forward checking
  - Dynamic rearrangement forward checking

Constraint Processing II & Waltz: The 4 Teachers problem

# **CONSTRAINT PROCESSING: PROBLEM REPRESENTATION**

# Constraint Processing

- Problem representation:

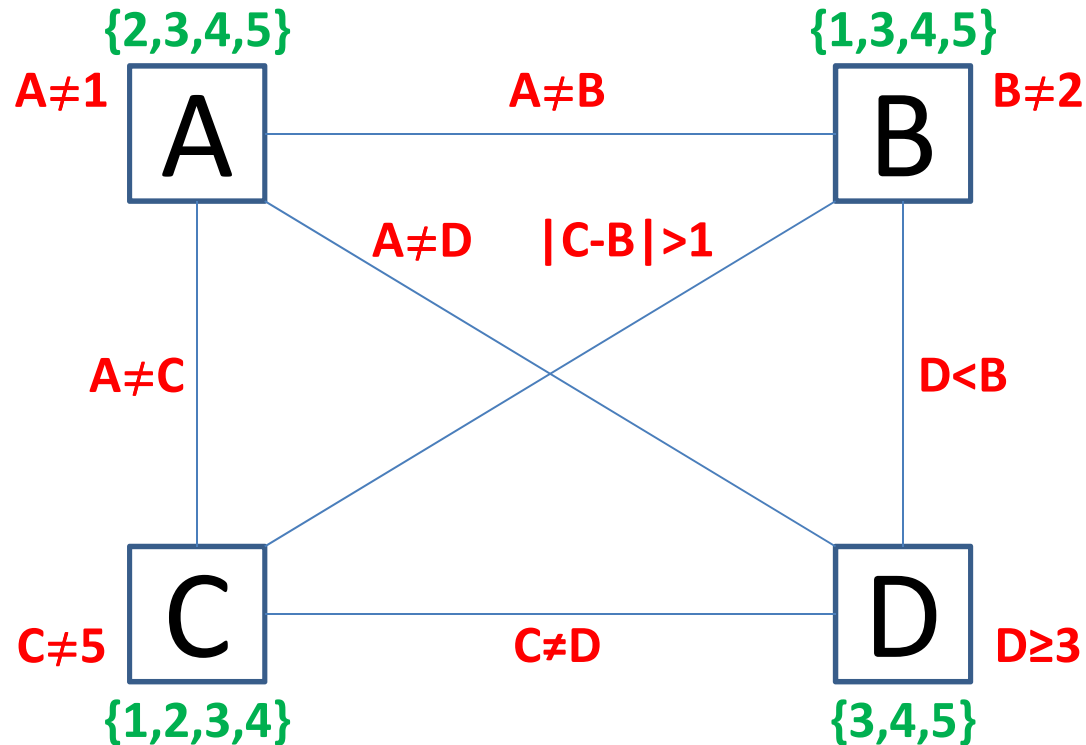


MiniMax & Constraint Processing: The 4 Houses problem

# **CONSTRAINT PROCESSING: PROBLEM OPTIMIZATION**

# Problem Optimization

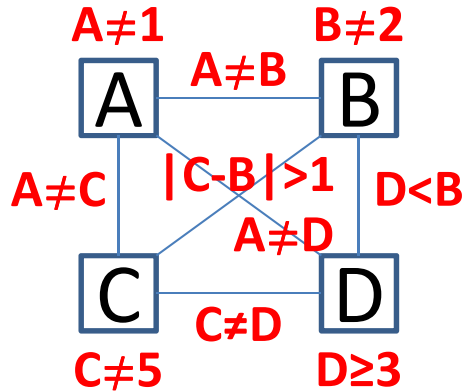
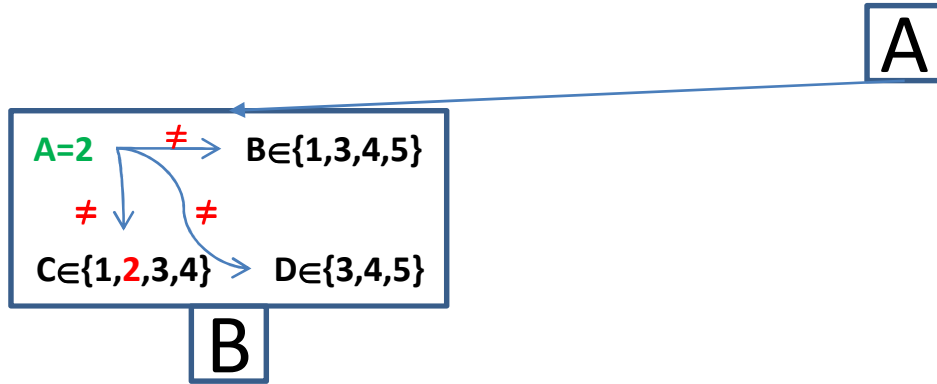
- Problem optimization:



MiniMax & Constraint Processing: The 4 Houses problem

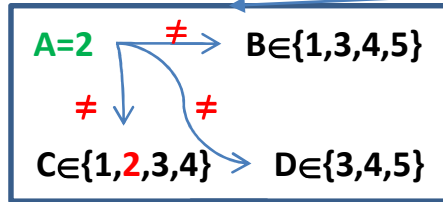
# **CONSTRAINT PROCESSING: FORWARD CHECKING**

# Forward Checking

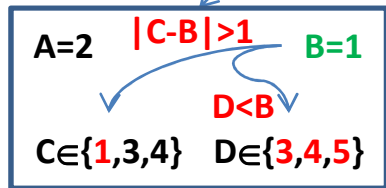


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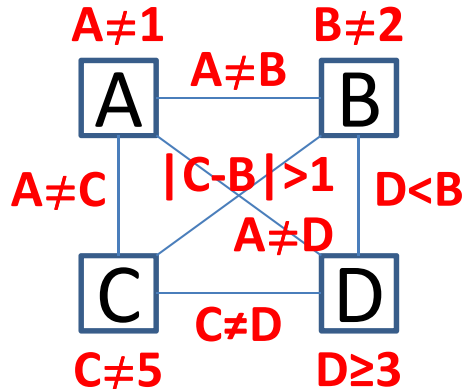
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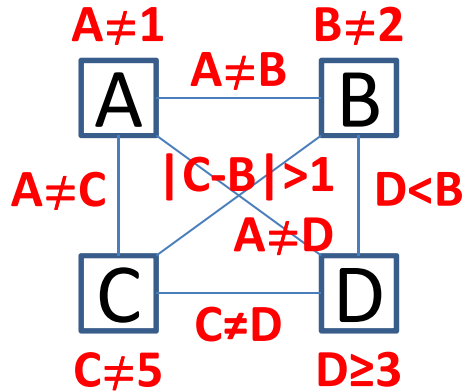
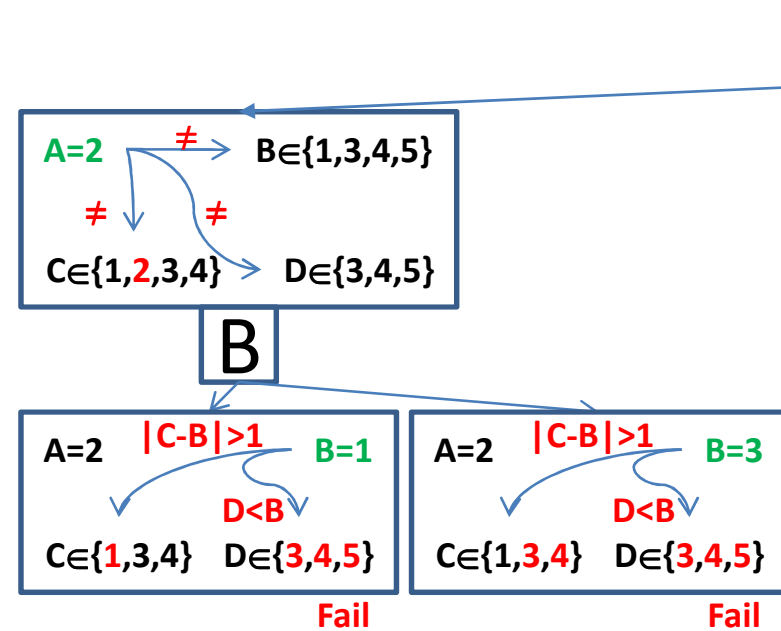
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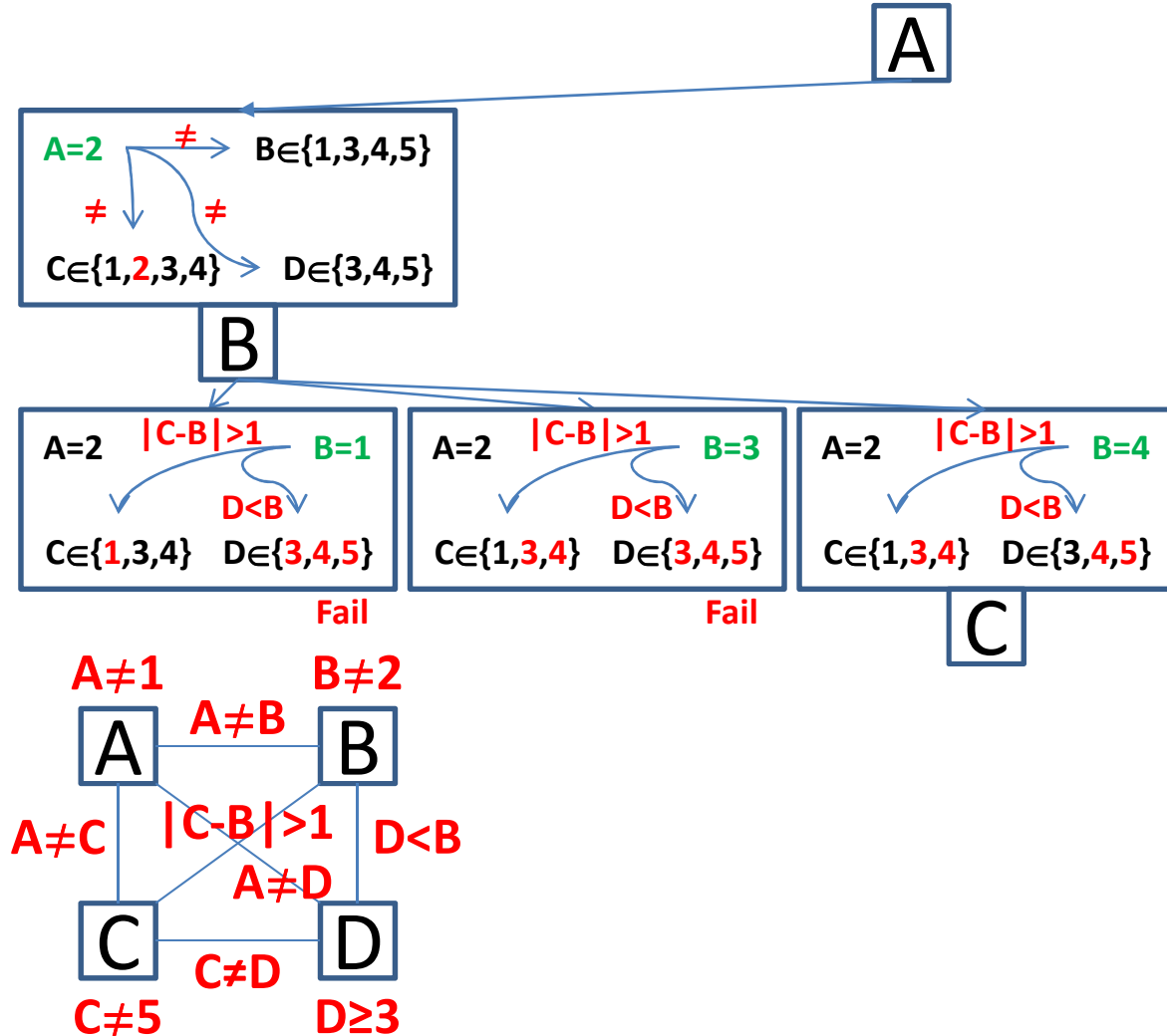
Fail



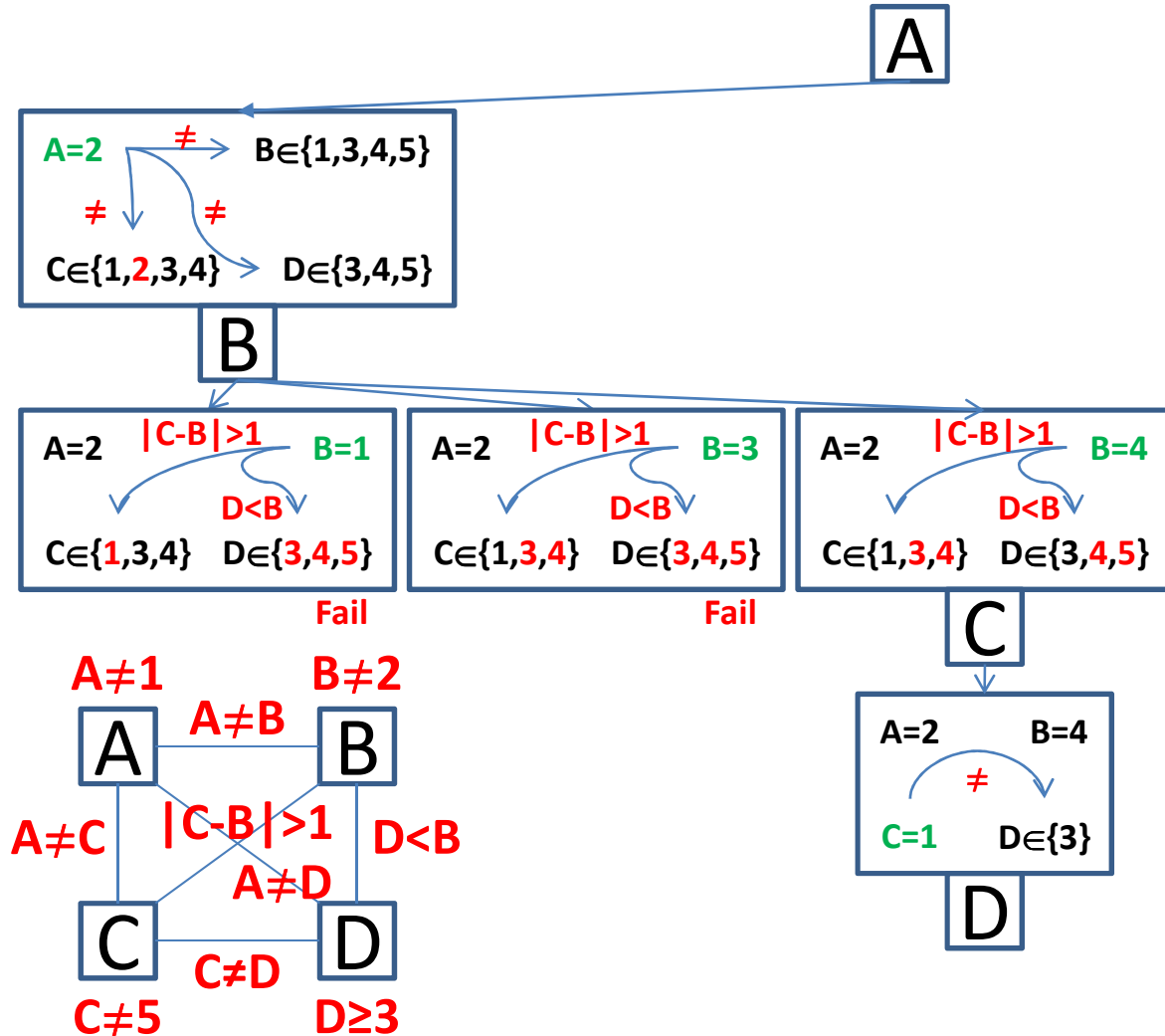
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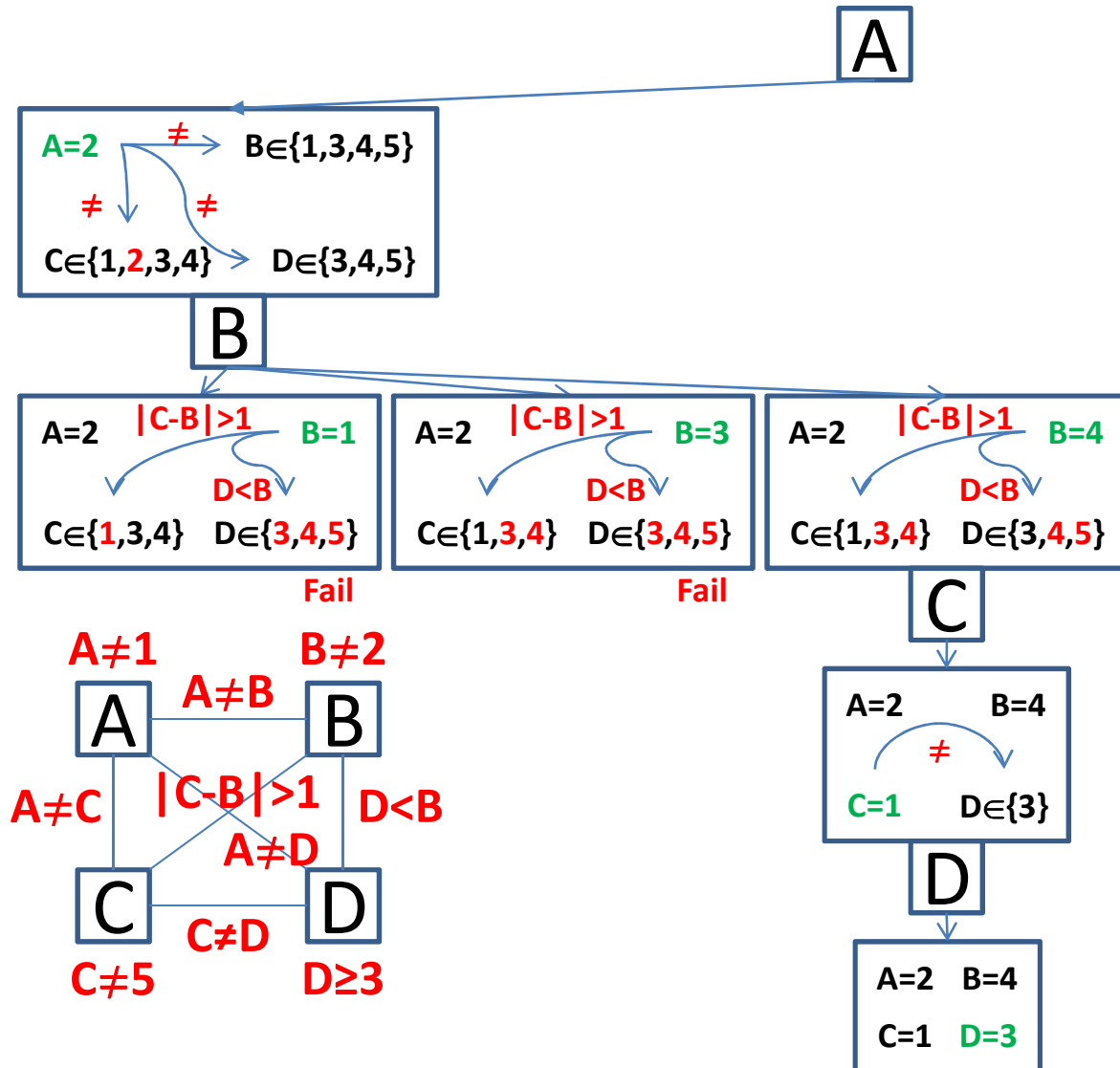
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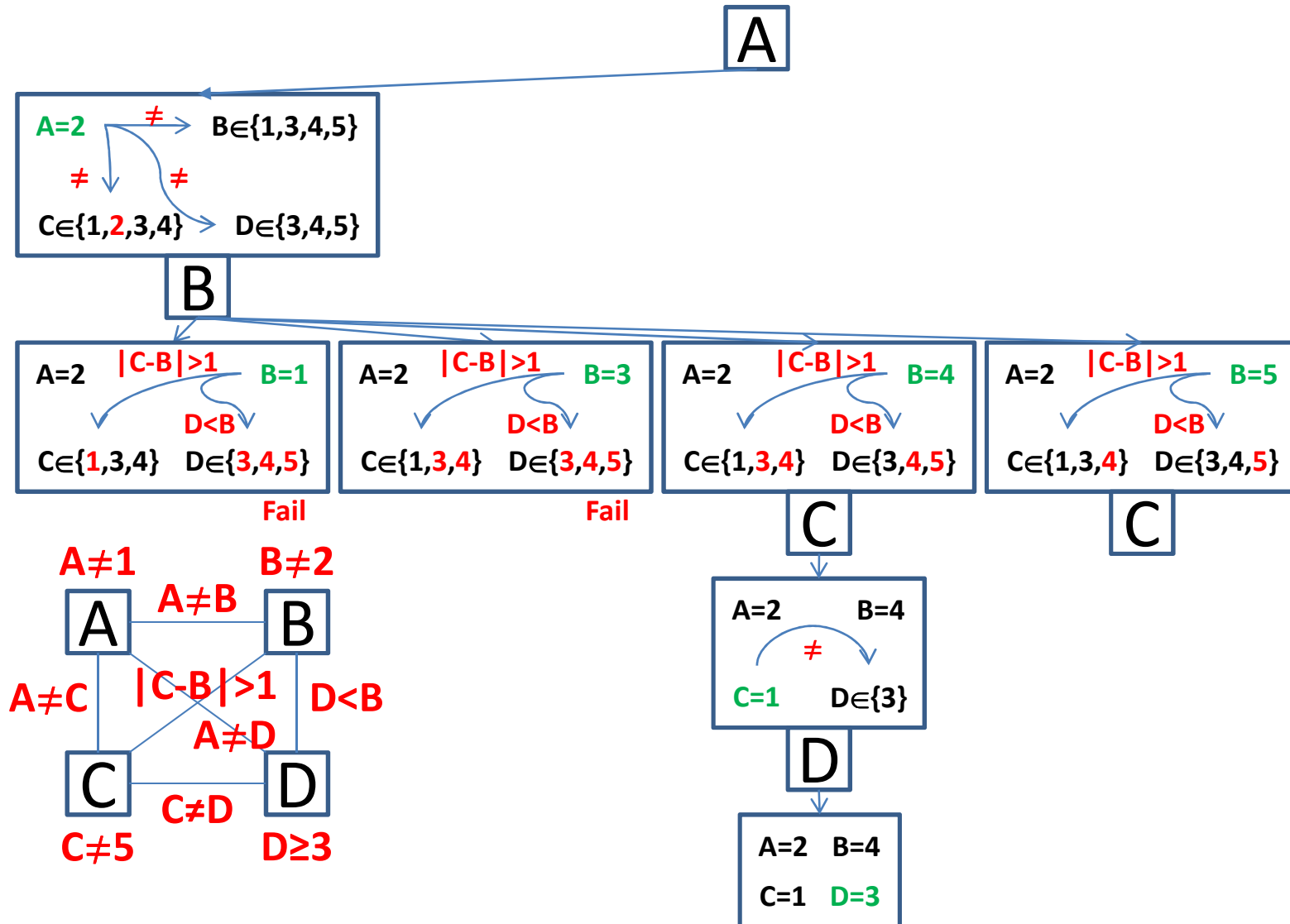
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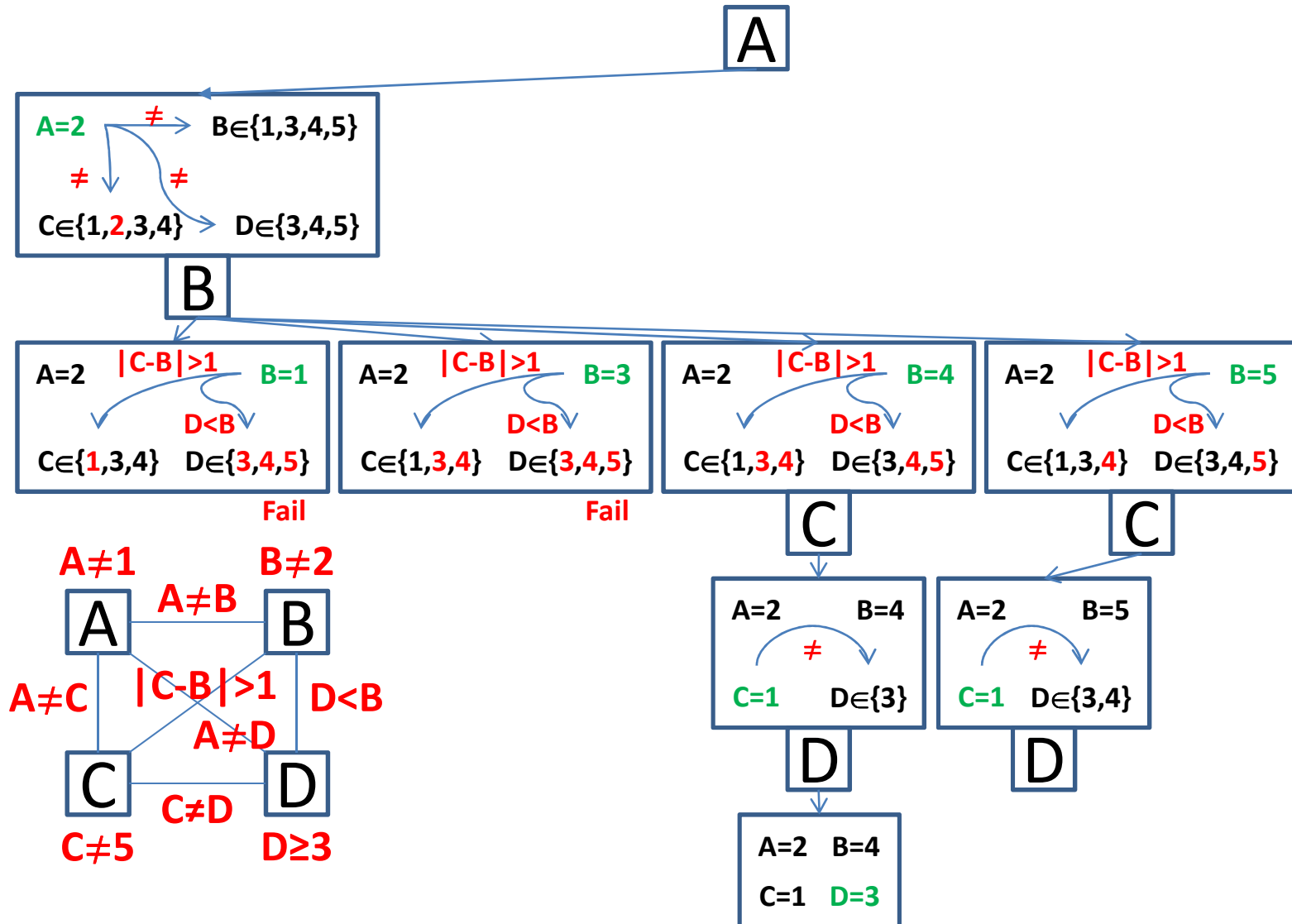
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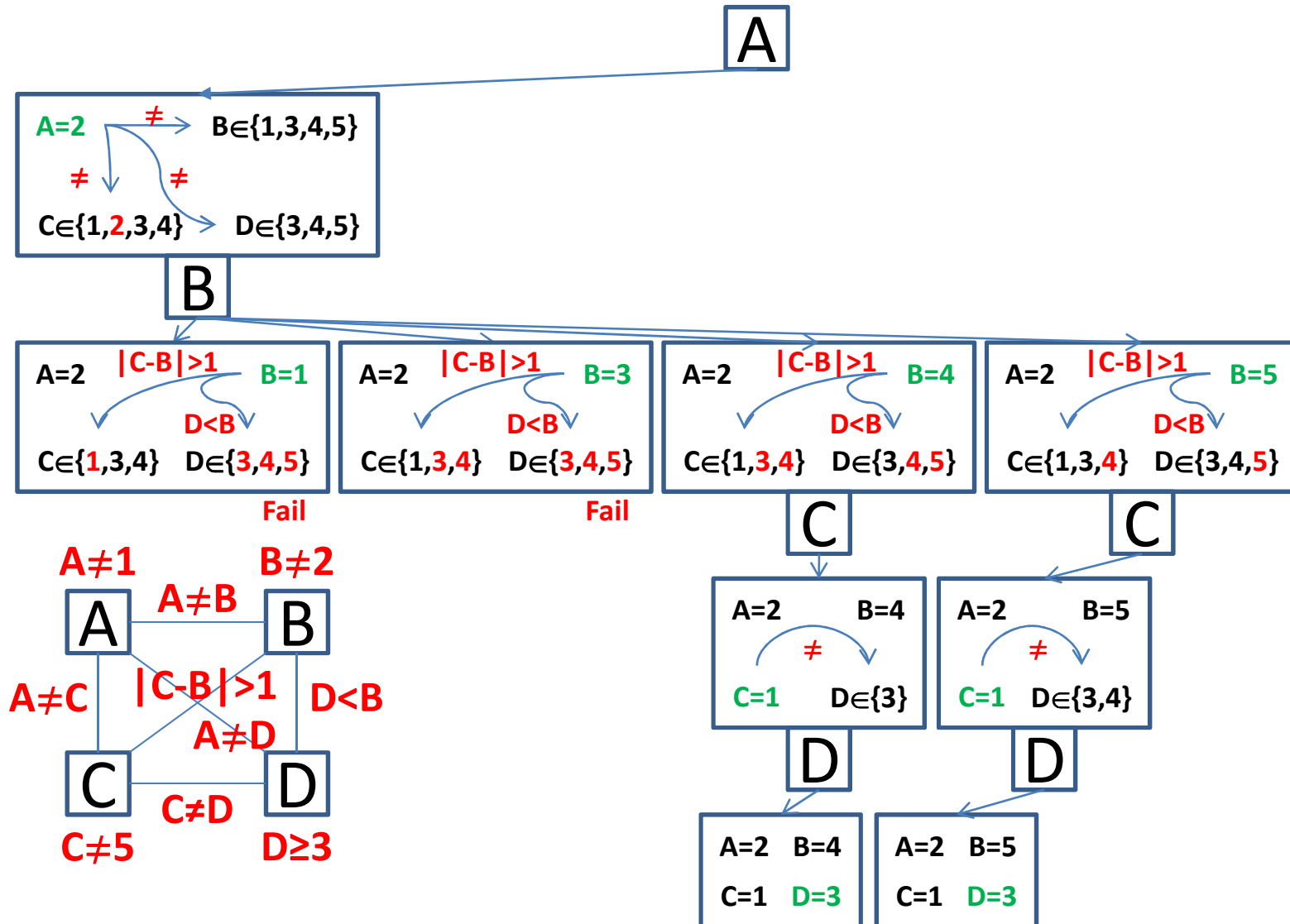
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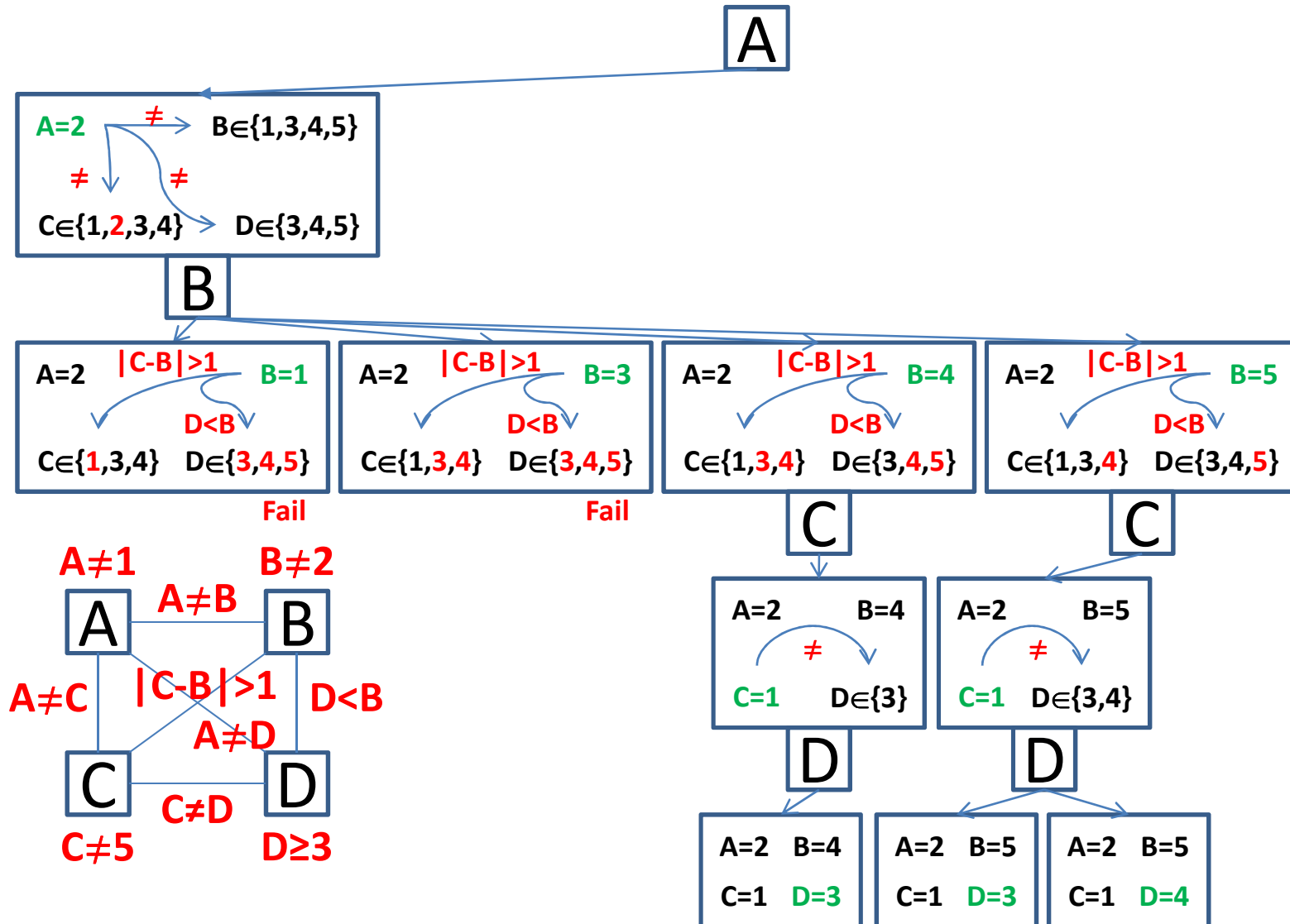
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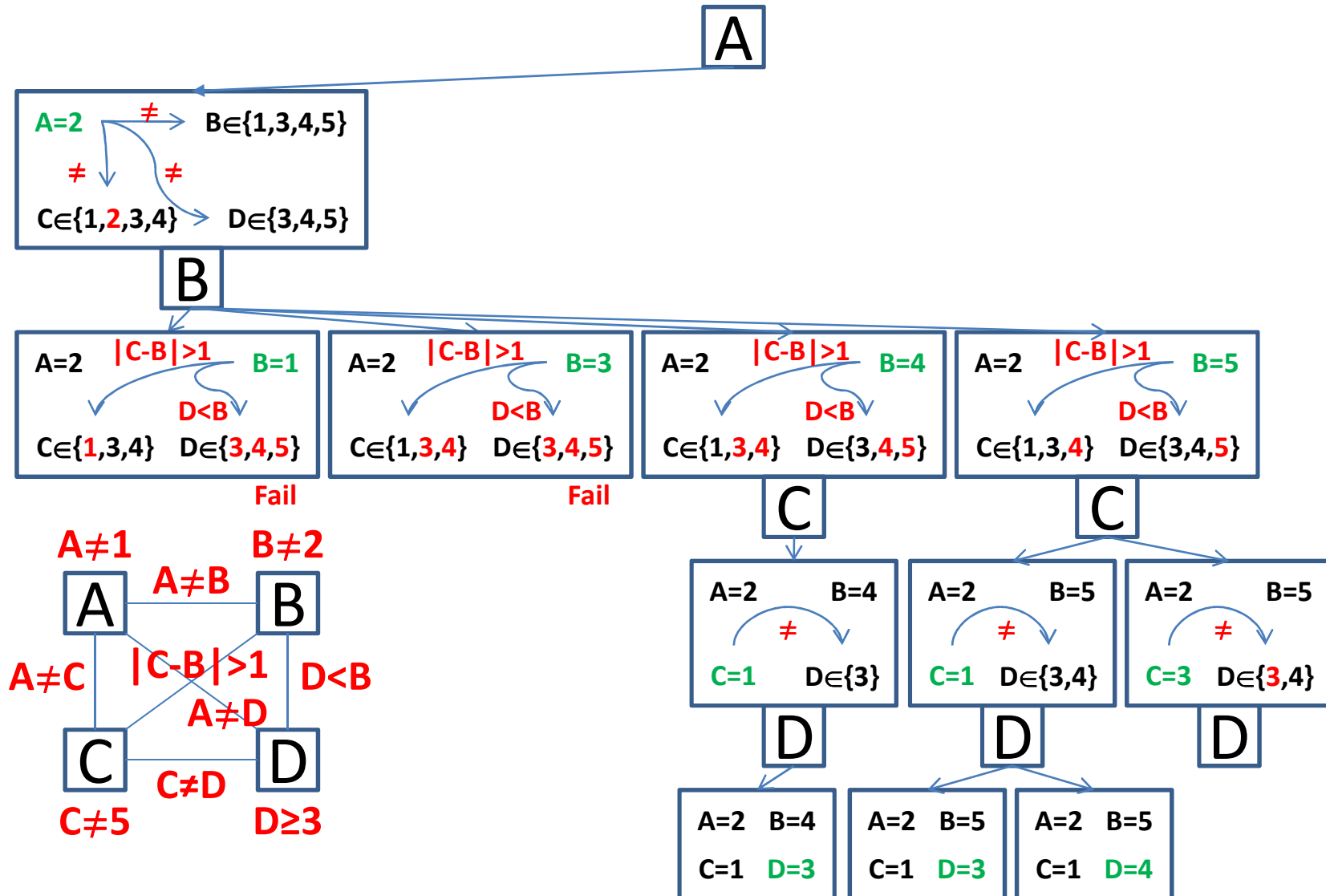
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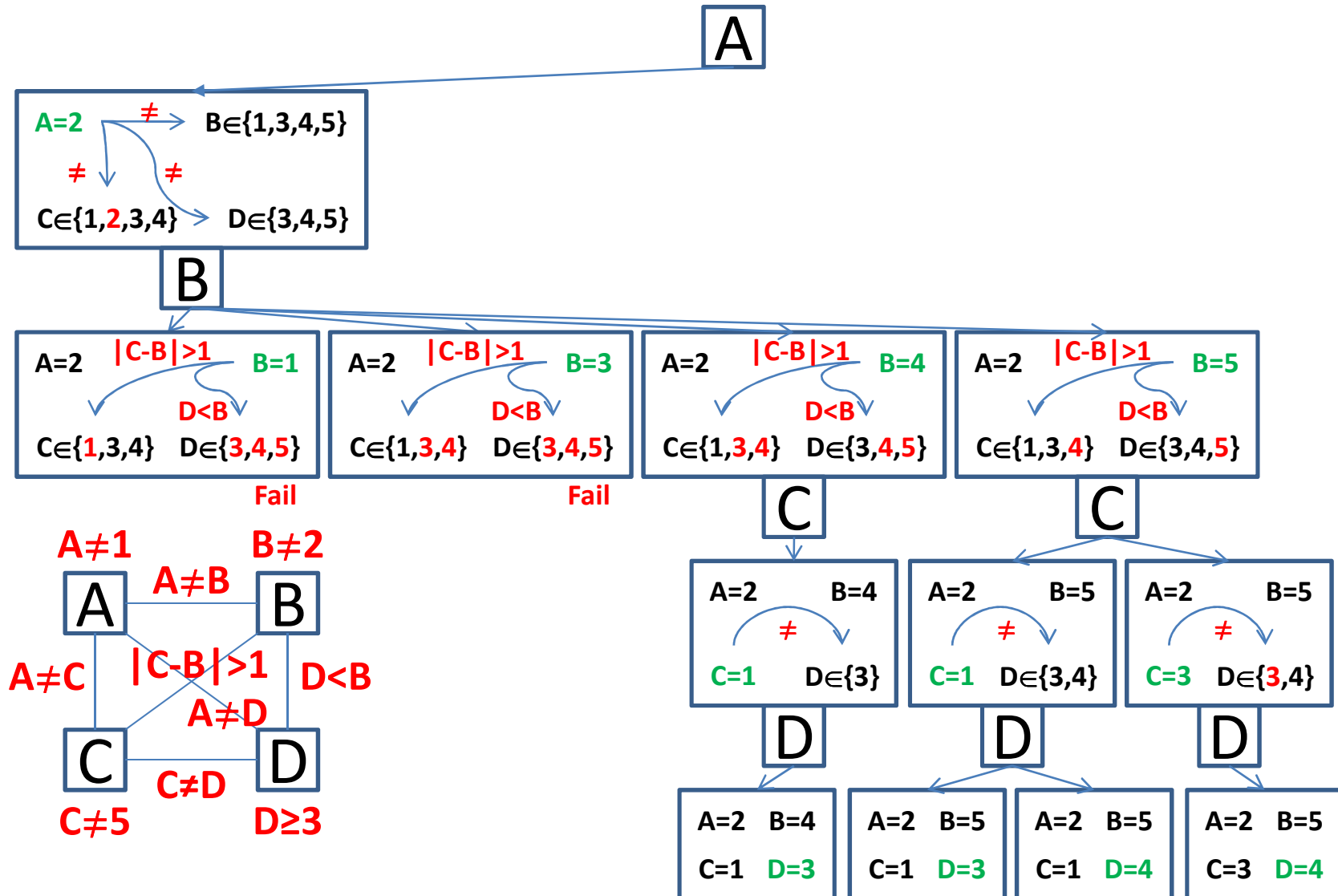
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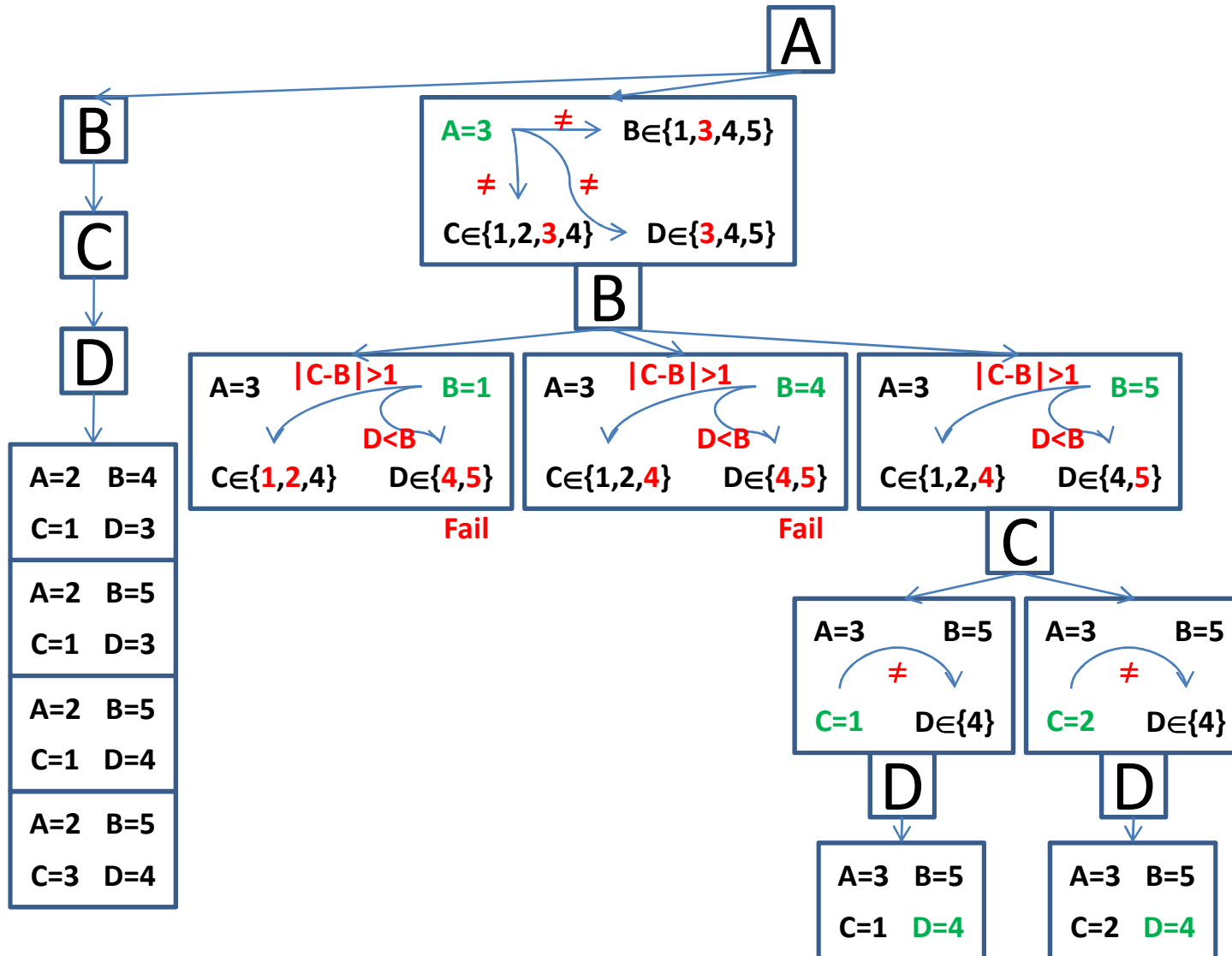
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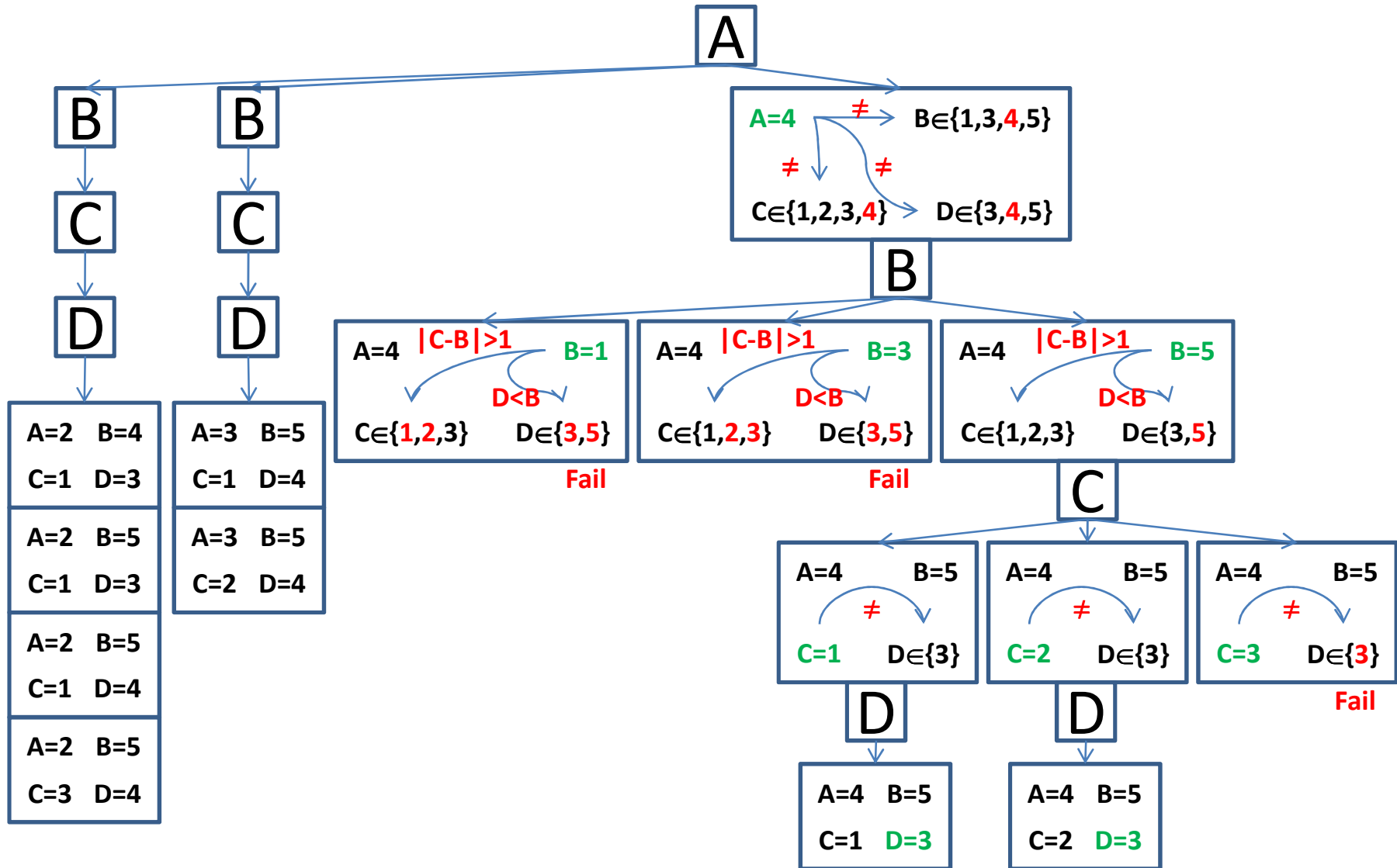
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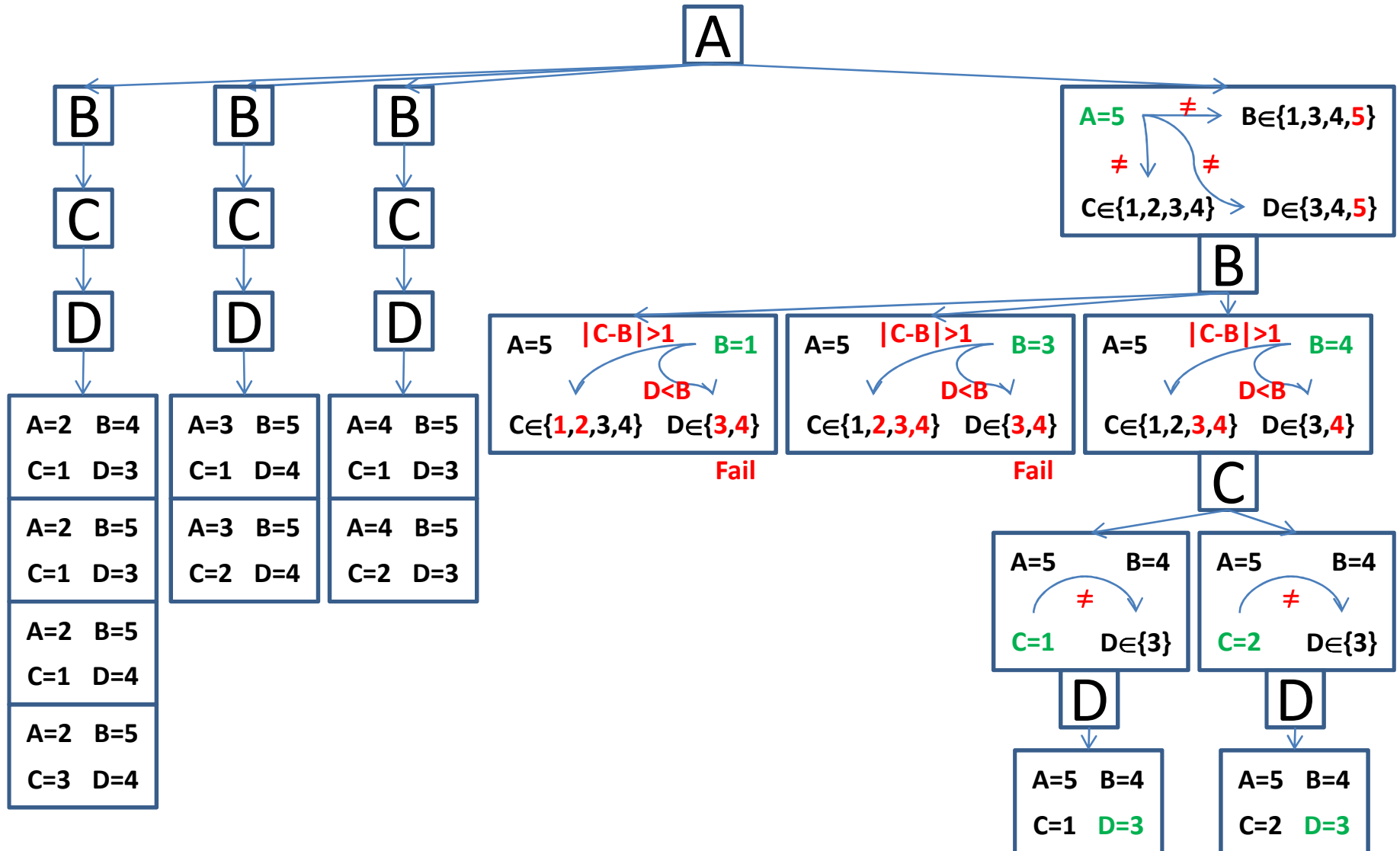
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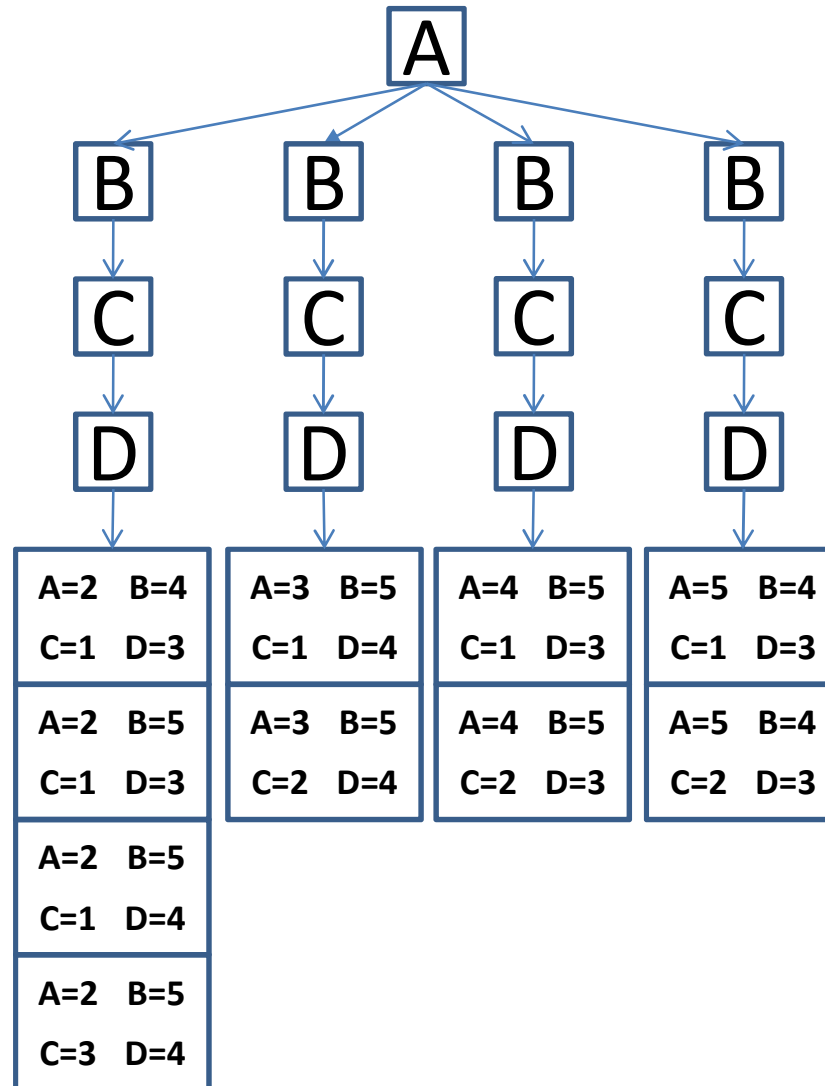
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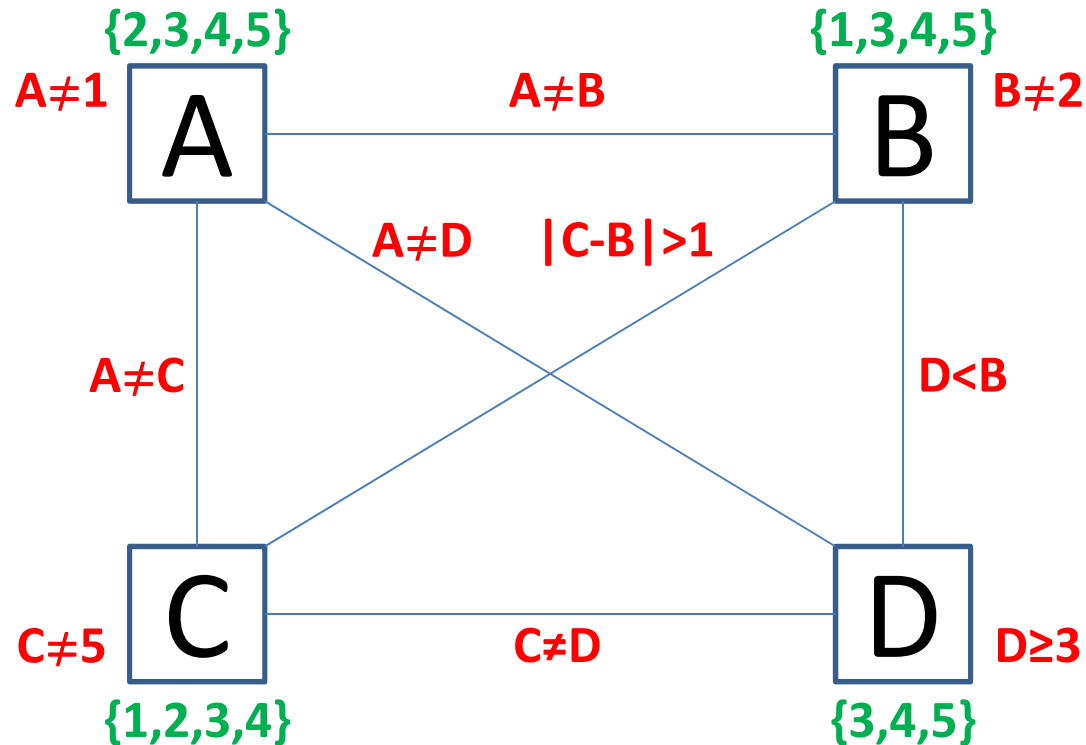
# Forward Checking



MiniMax & Constraint Processing: The 4 Houses problem

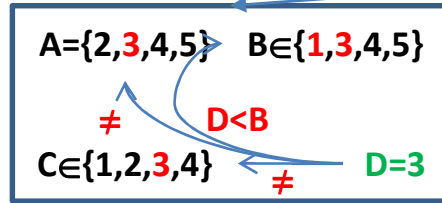
# **CONSTRAINT PROCESSING: DYNAMIC SEARCH REARRANGEMENT FC**

# Dynamic Search Rearrangement FC



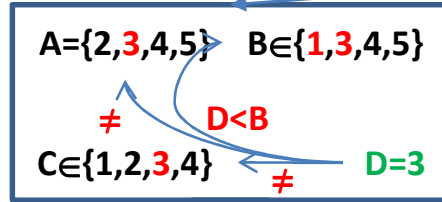
# Dynamic Search Rearrangement FC

**D** Smallest Domain

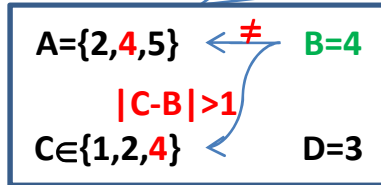


# Dynamic Search Rearrangement FC

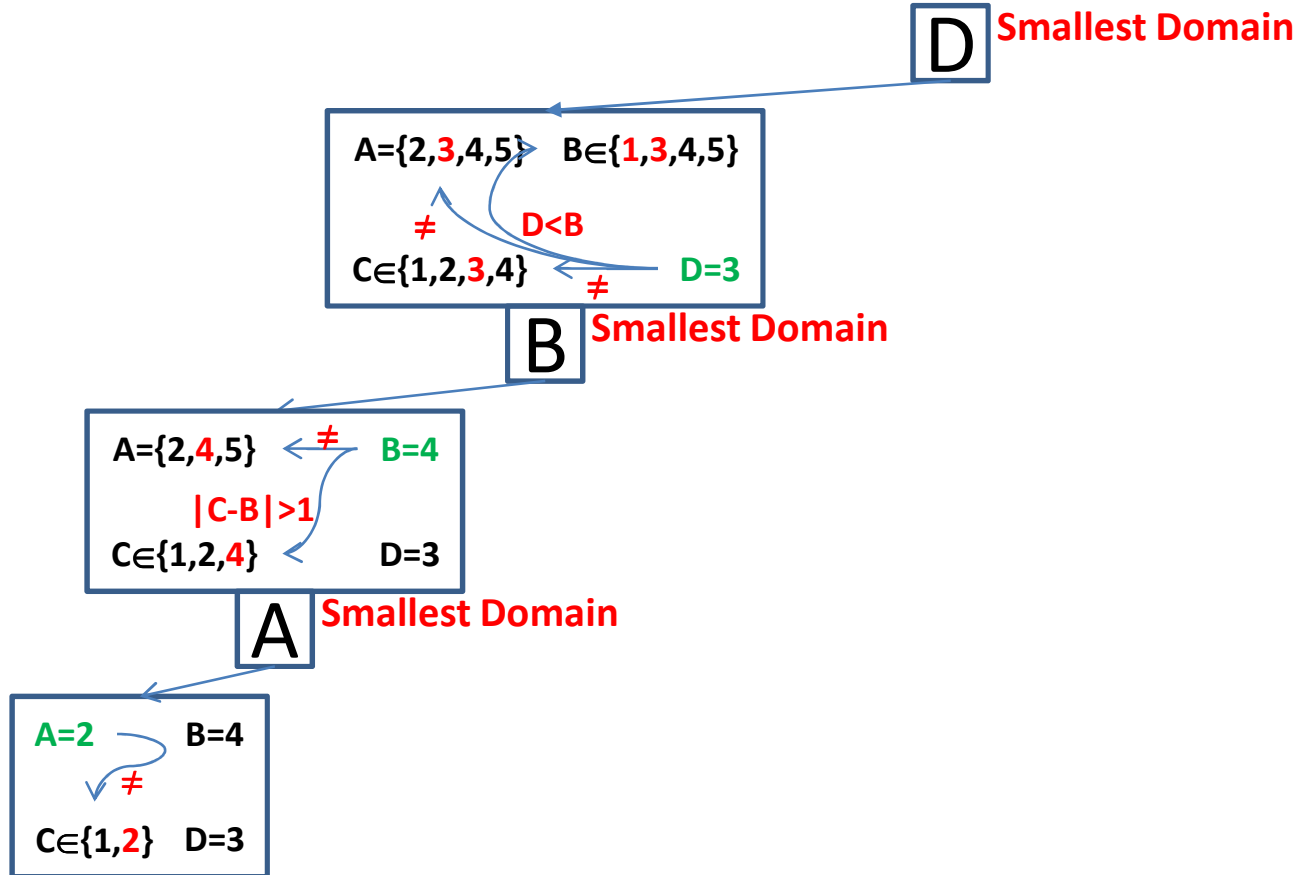
**D** Smallest Domain



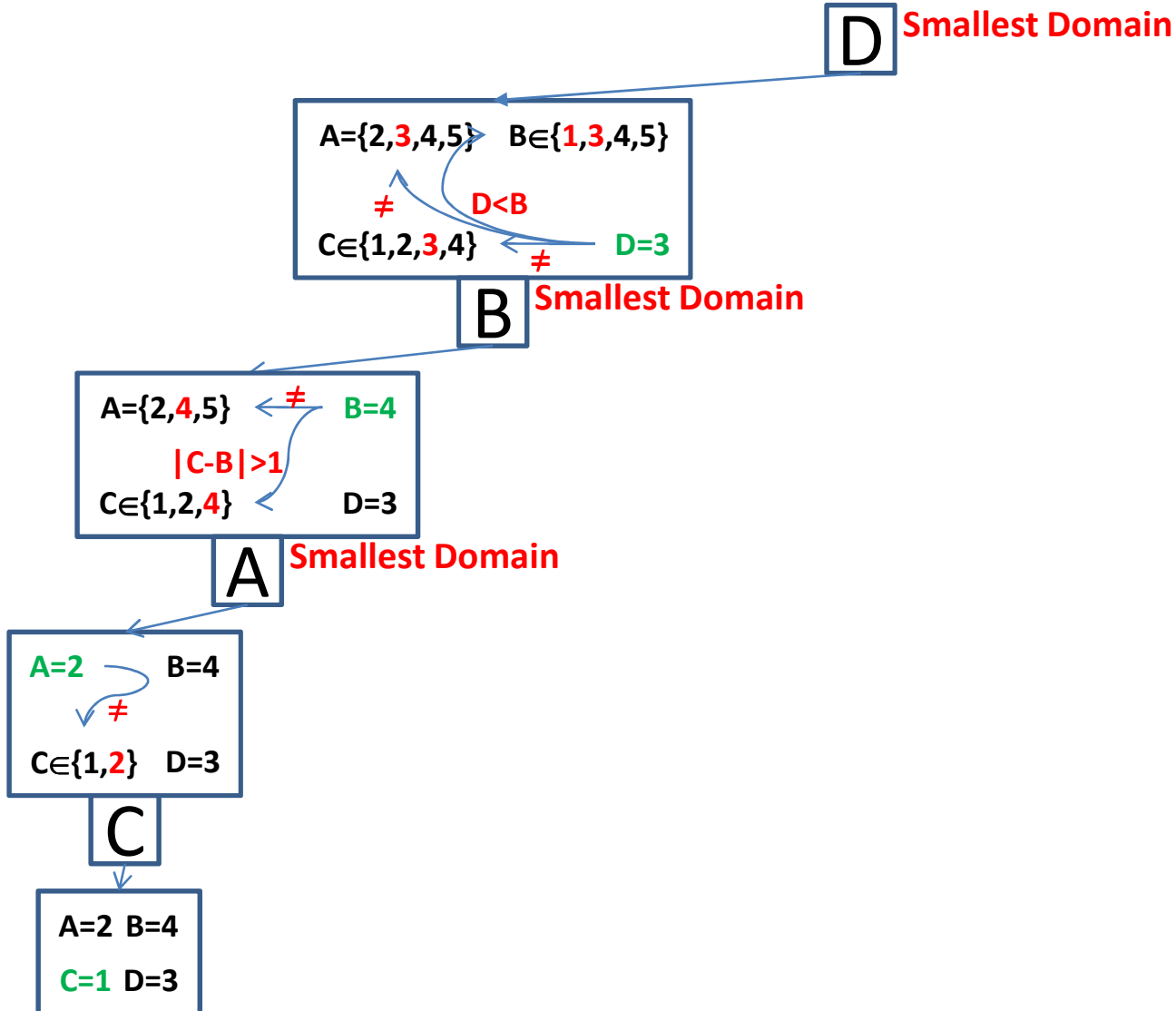
**B** Smallest Domain



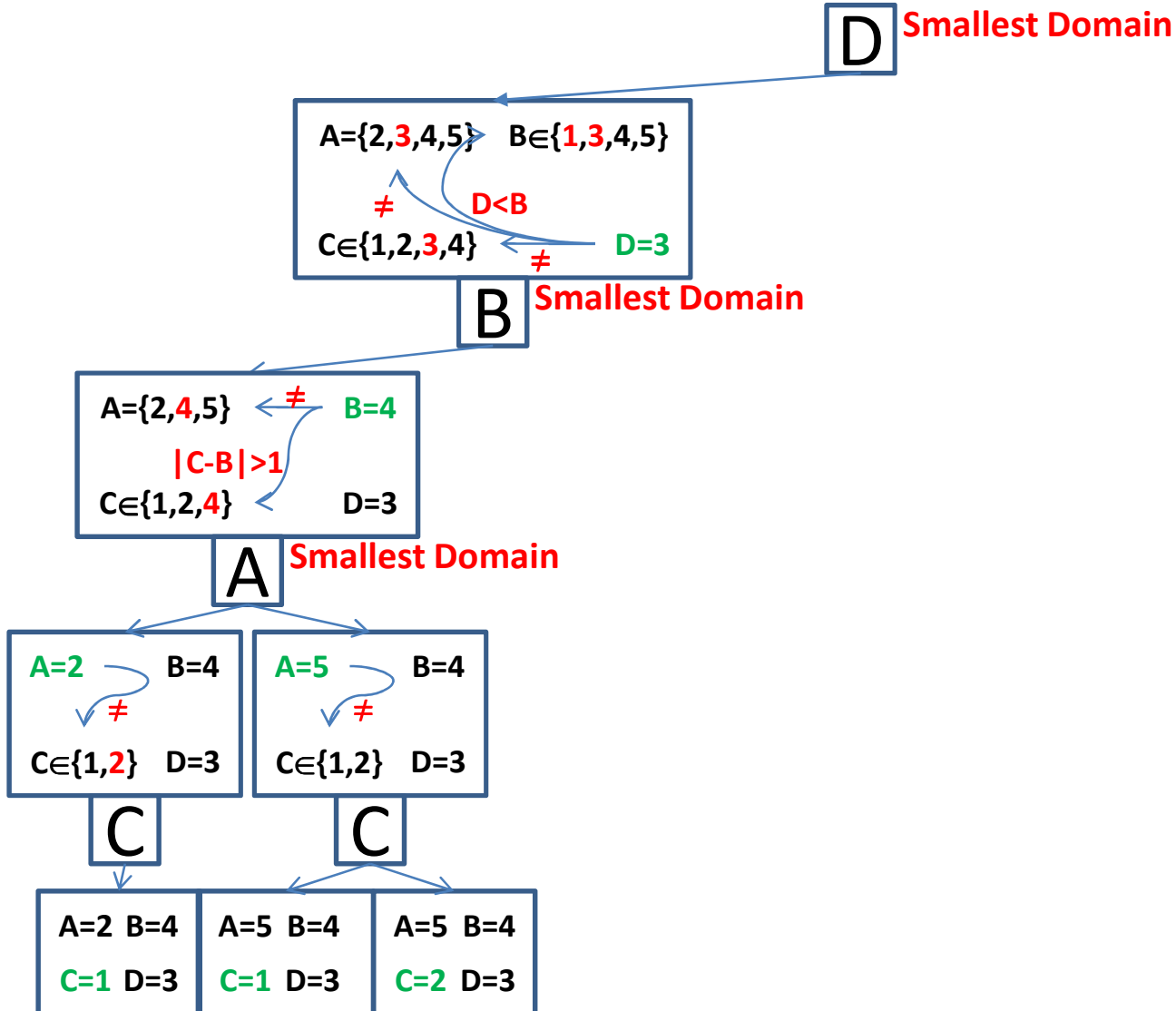
# Dynamic Search Rearrangement FC



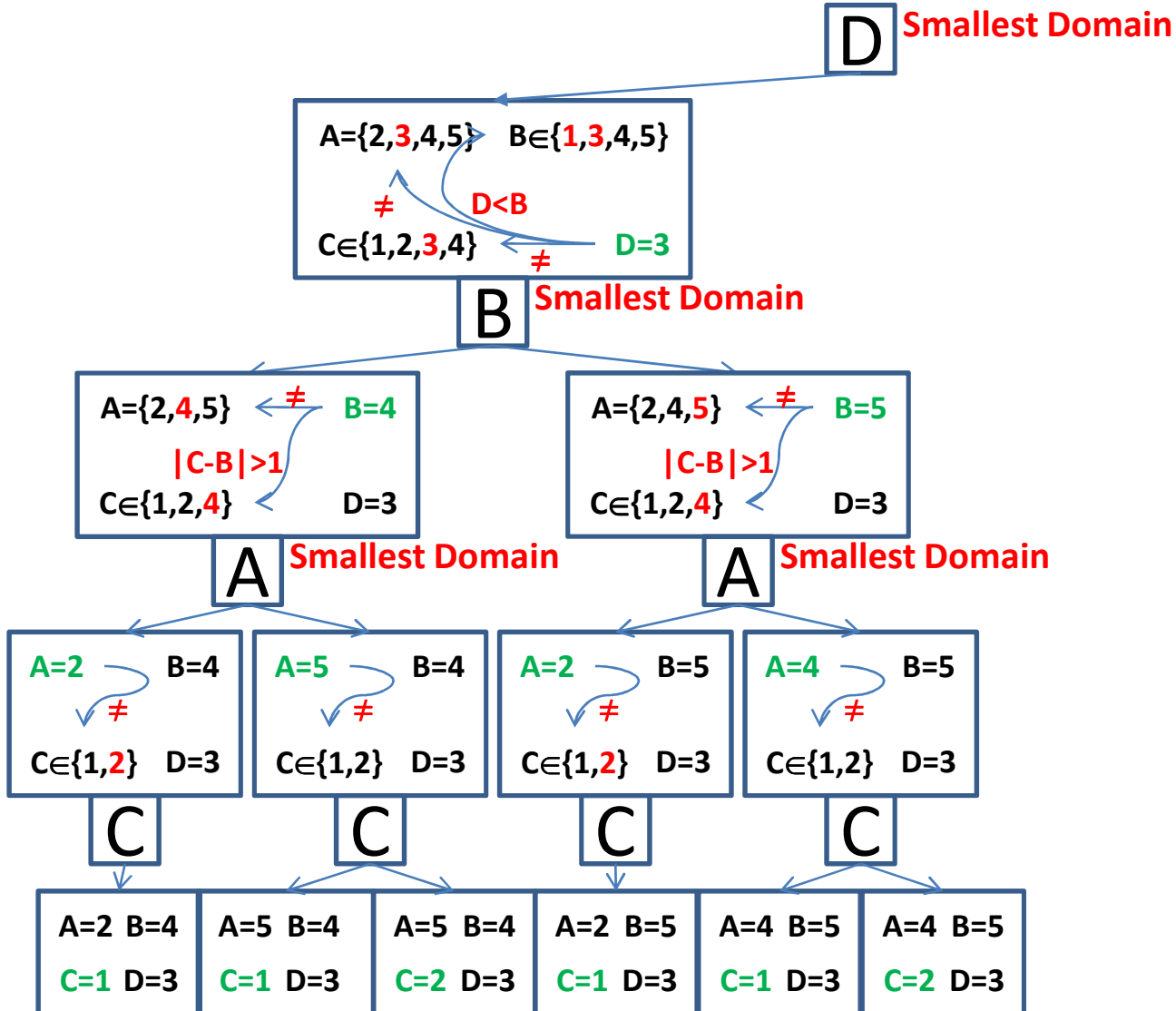
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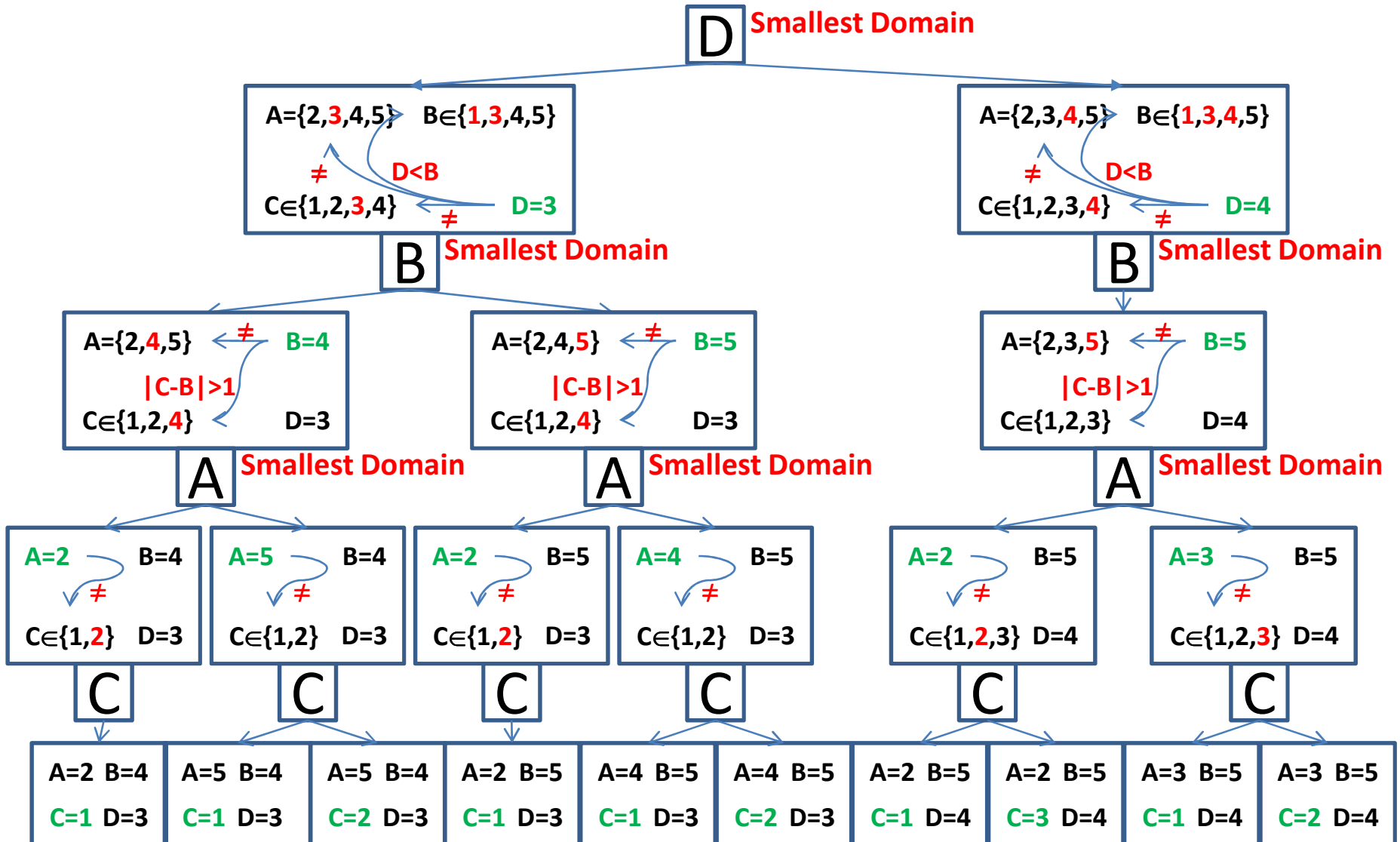
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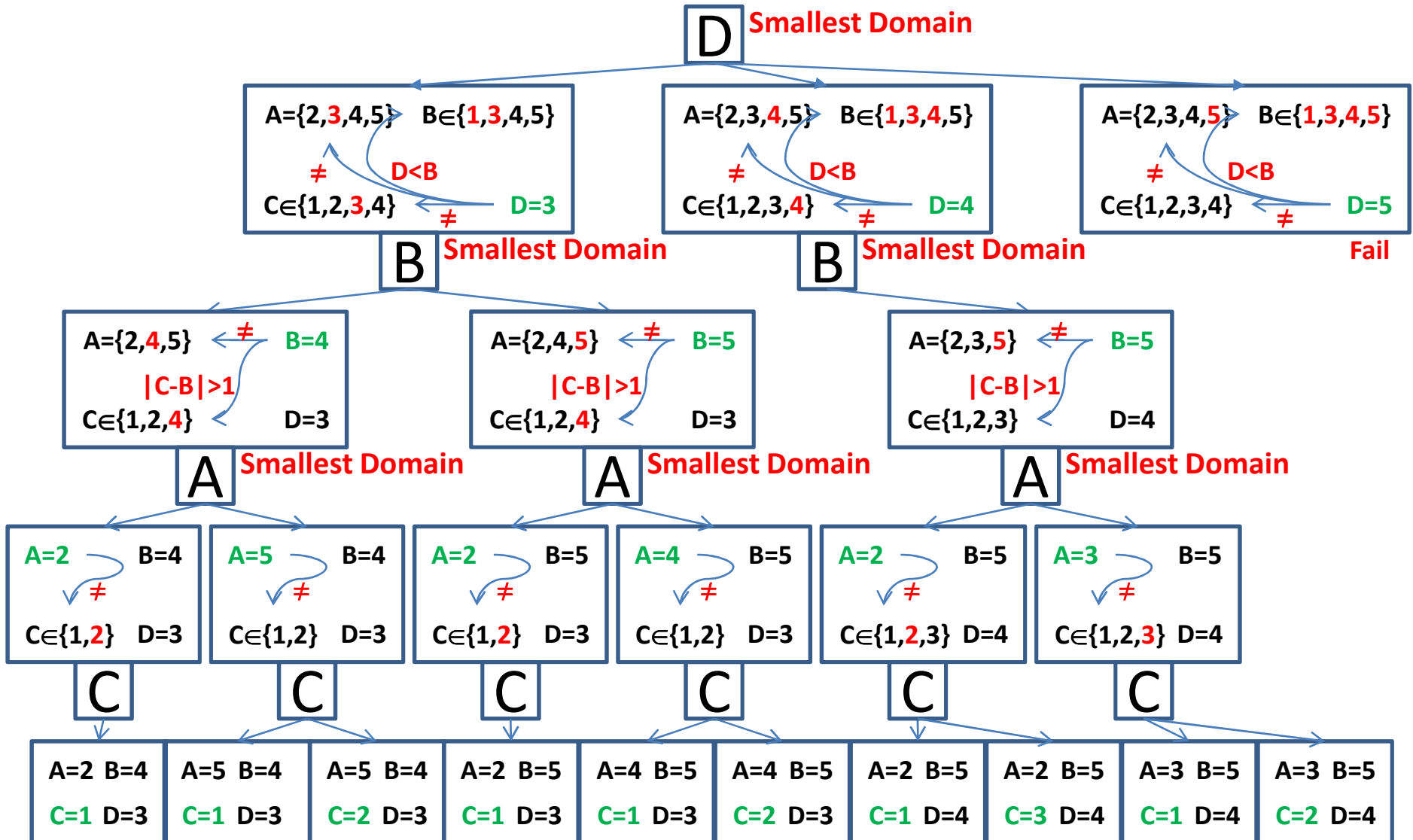
# Dynamic Search Rearrangement FC



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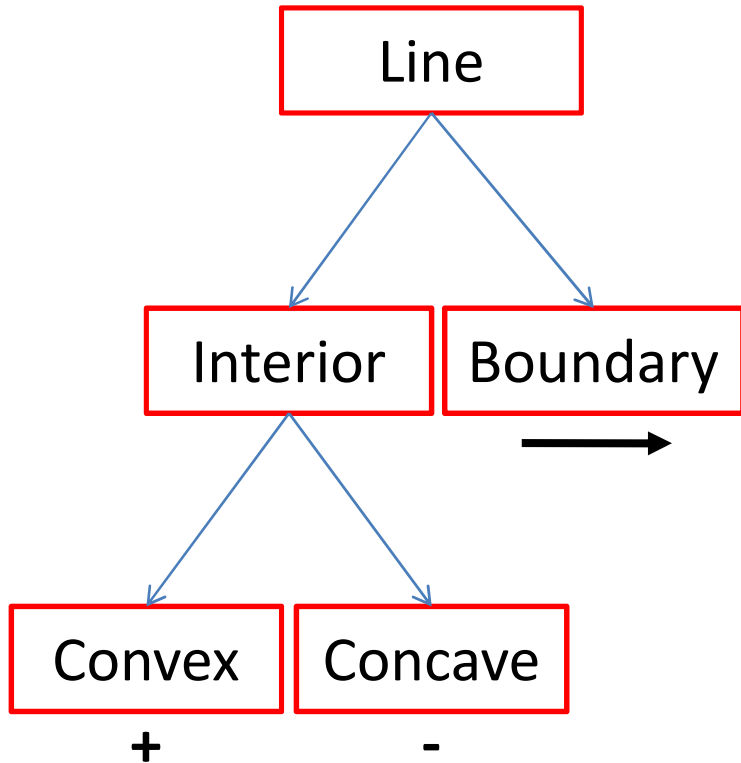
# Exercises: Artificial Intelligence

Constraint Processing II & Waltz:  
Waltz I

Constraint Processing II & Waltz: Waltz I

# **INTRODUCTION WALTZ**

# Introduction Waltz



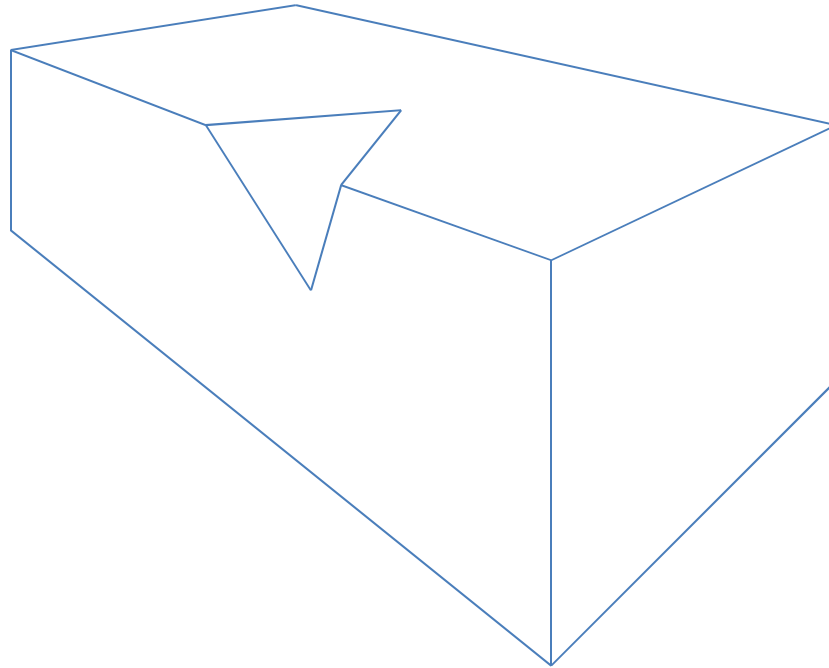
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|---|------|---|-------|
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Constraint Processing II & Waltz: Waltz I

# PROBLEM

# Problem

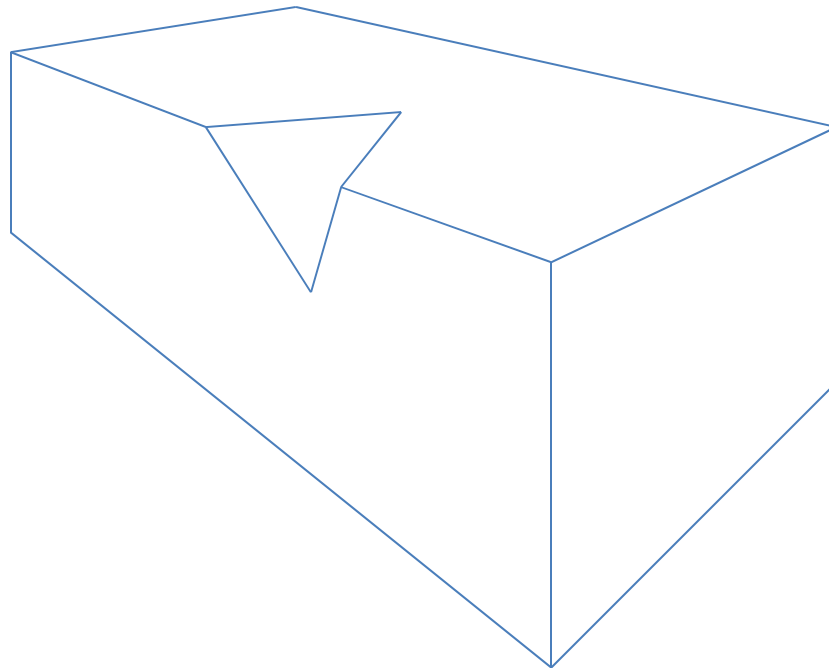
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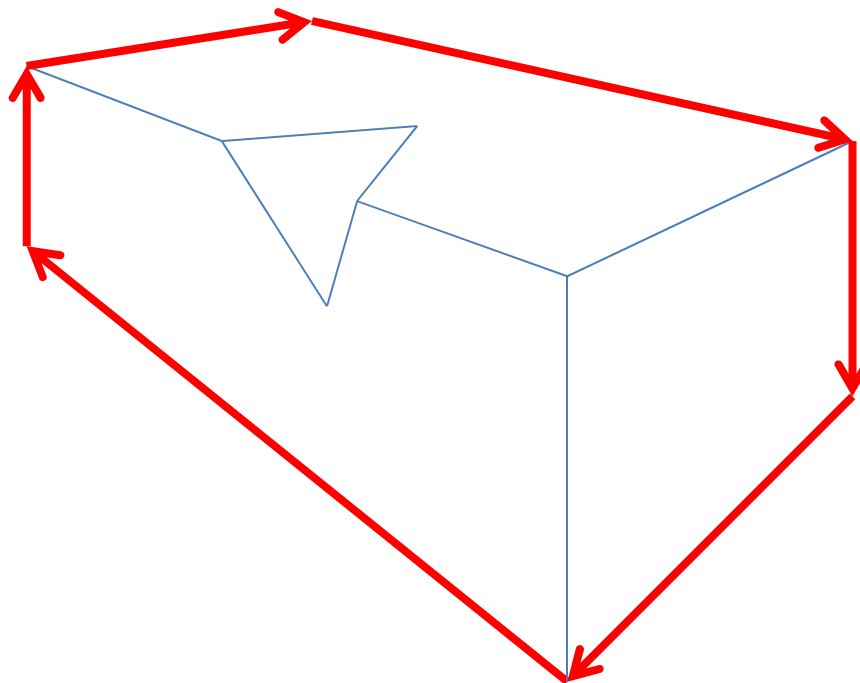
Constraint Processing II & Waltz: Waltz I

**SOLUTION**

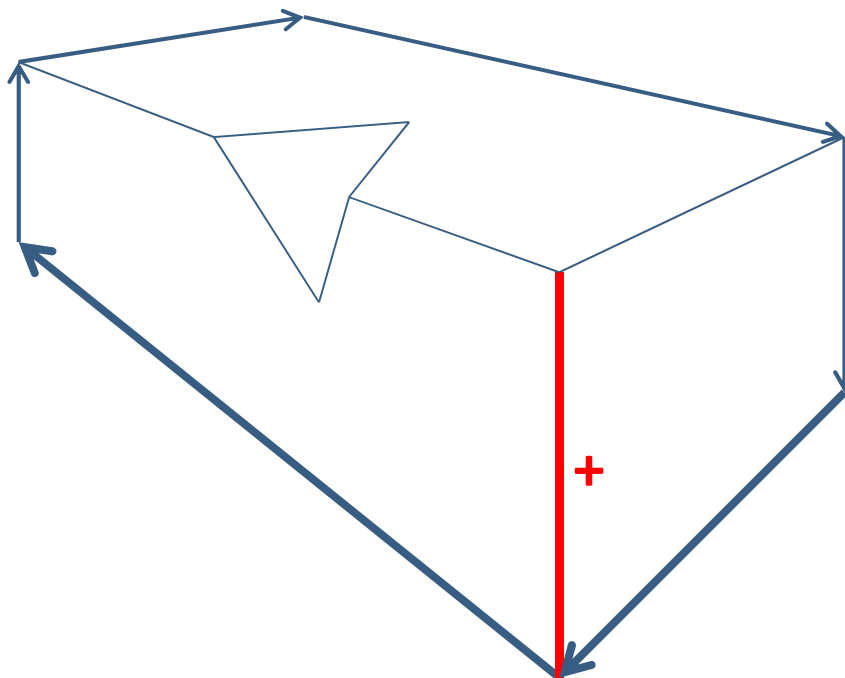
# Solution



# Solution

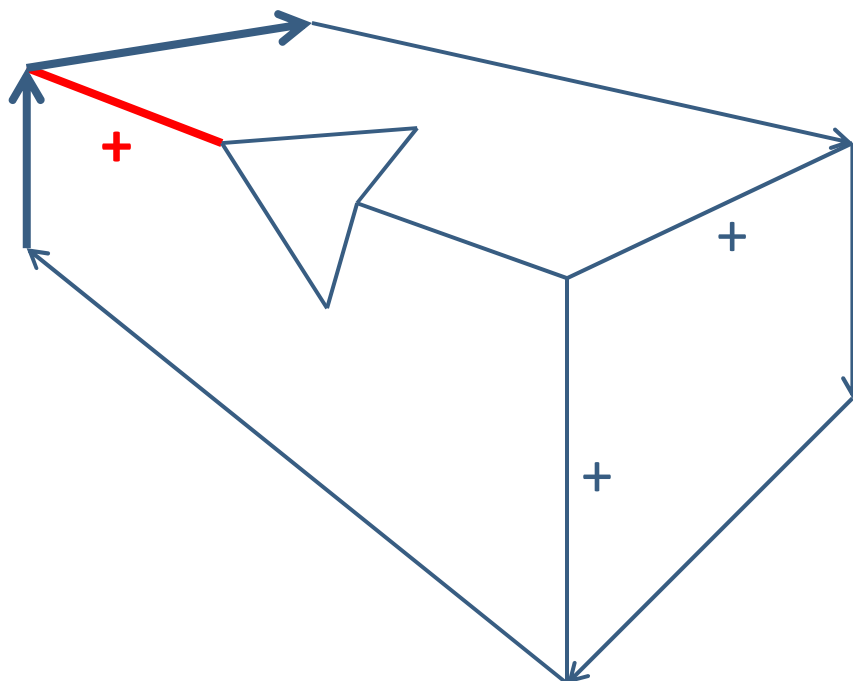


# Solution

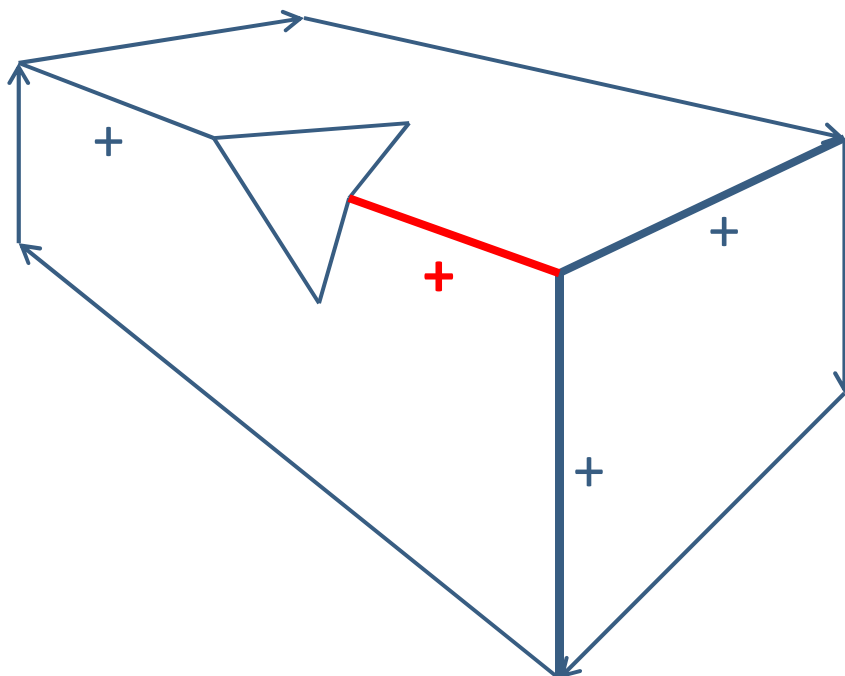




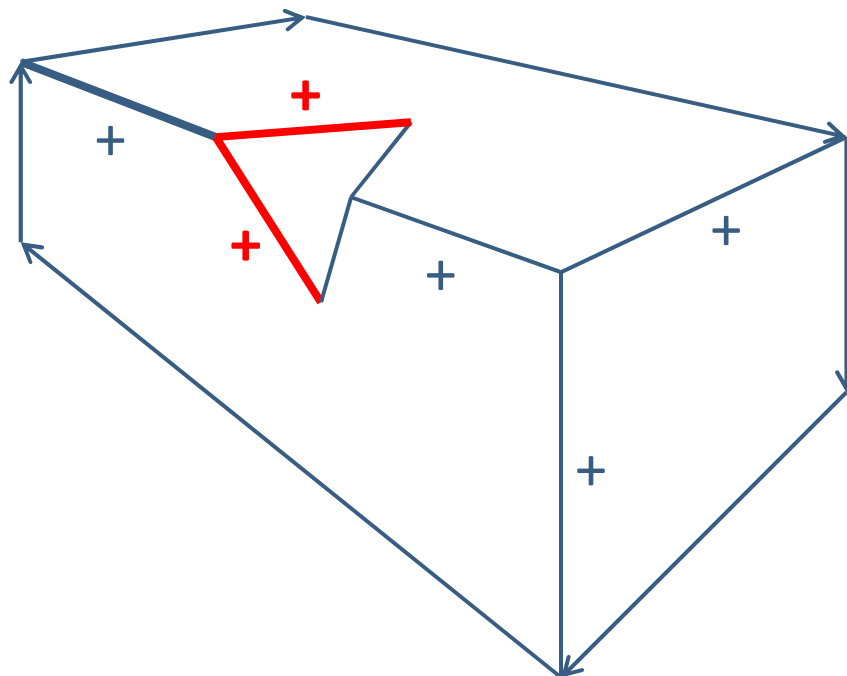
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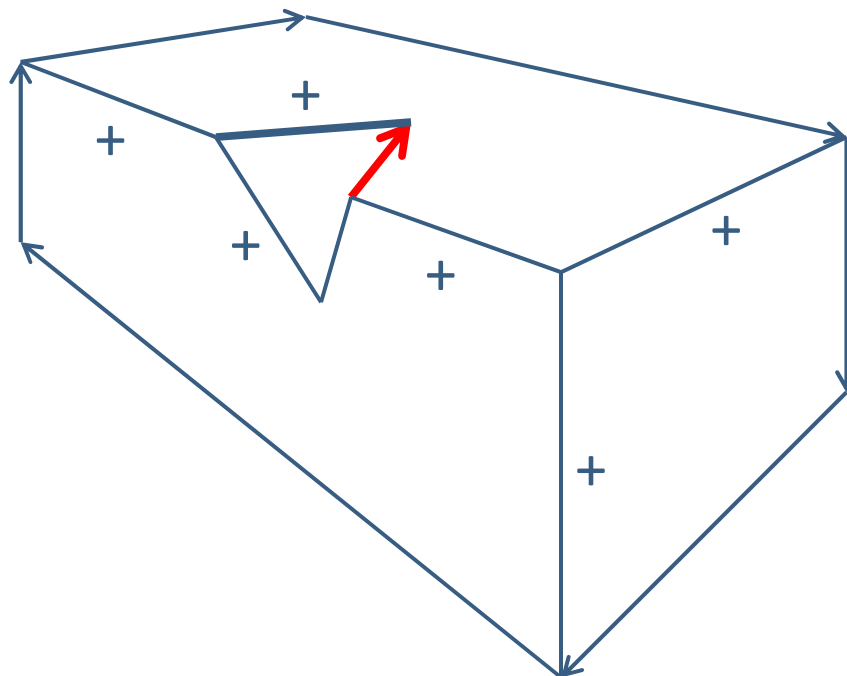
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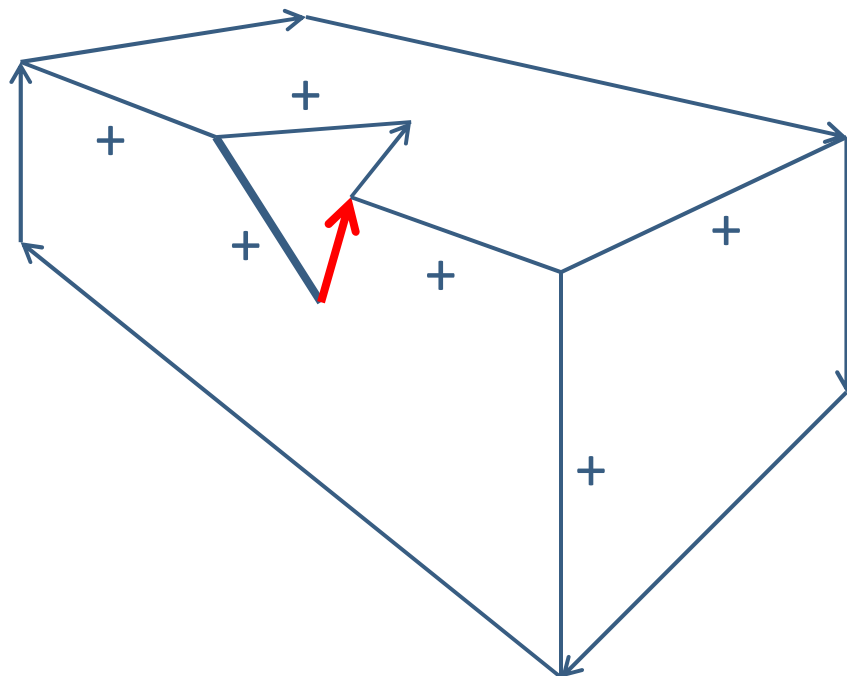
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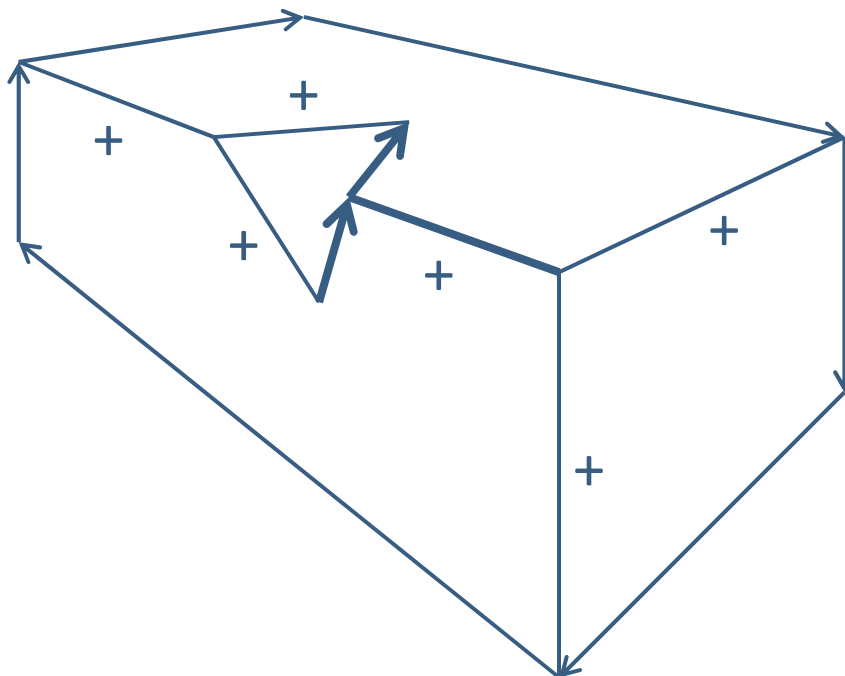
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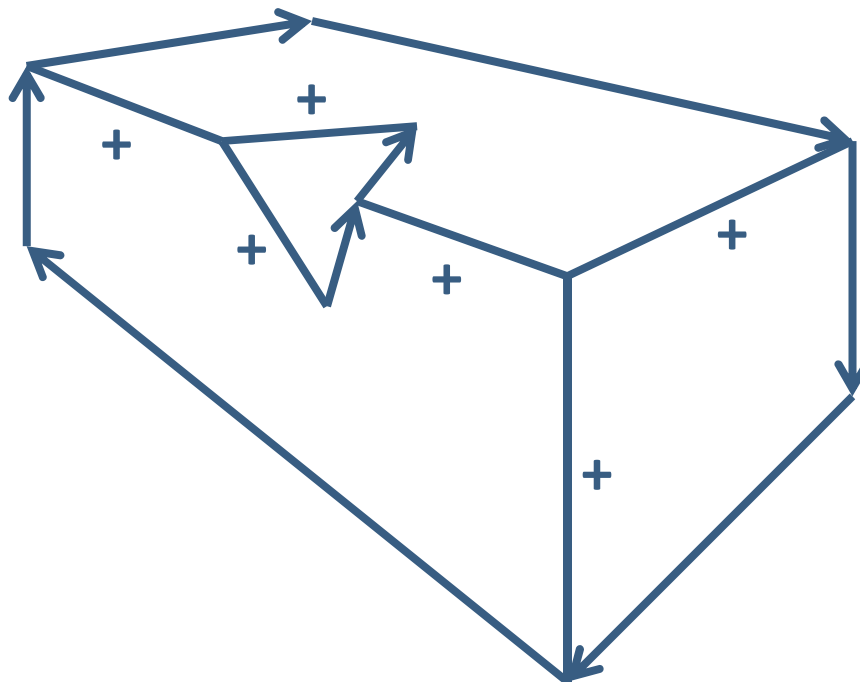
# Solution



# Solution



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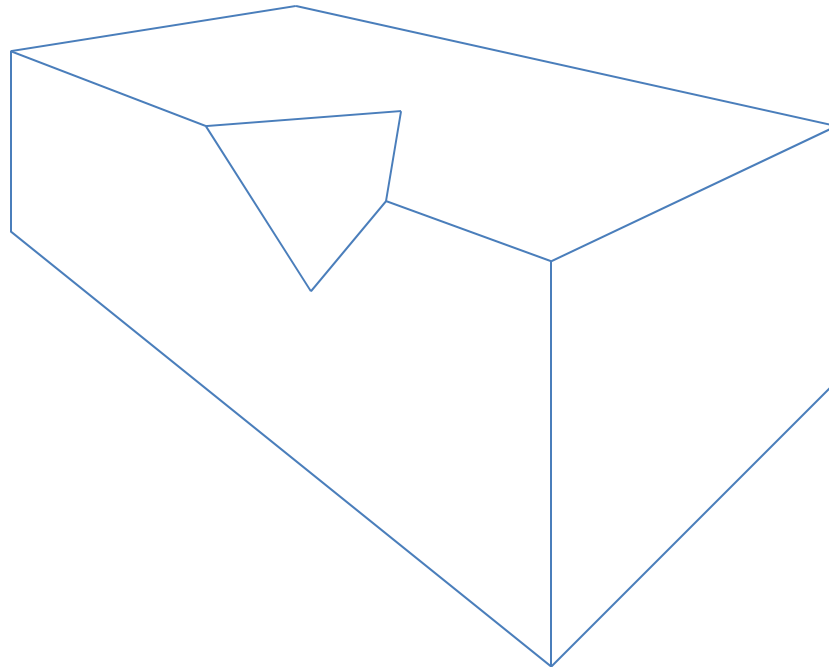


Constraint Processing II & Waltz: Waltz I

# **PROBLEM**

# Problem

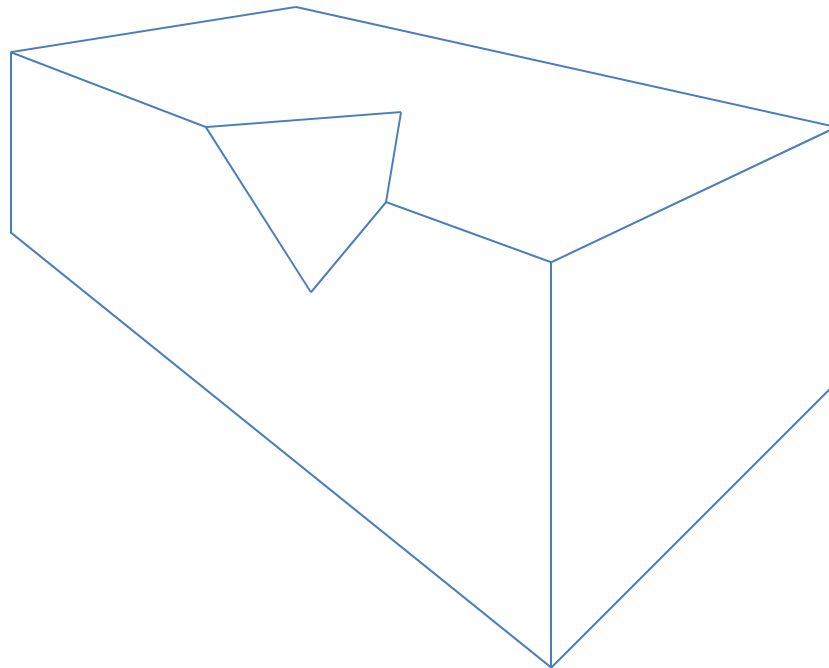
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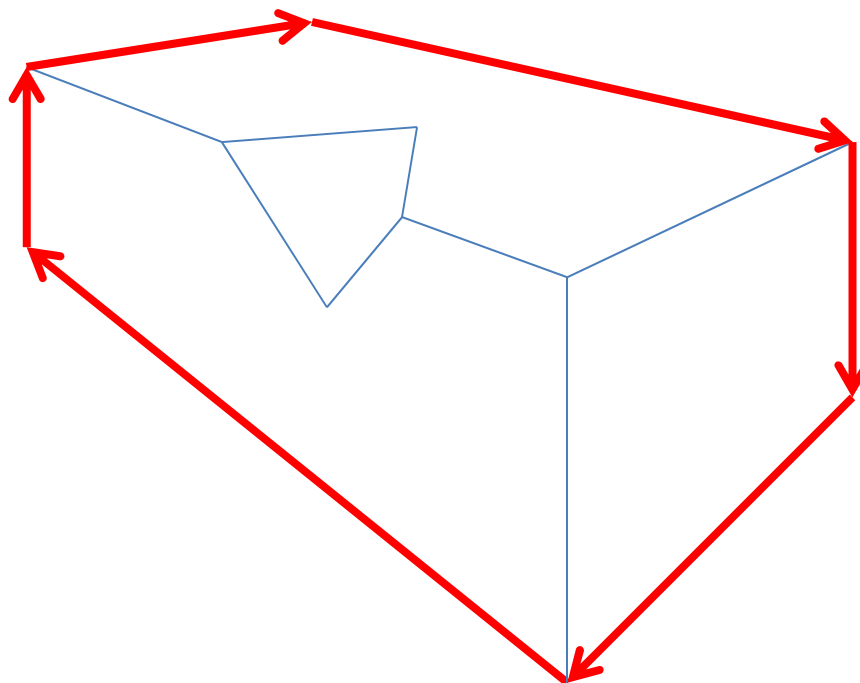
Constraint Processing II & Waltz: Waltz I

**SOLUTION**

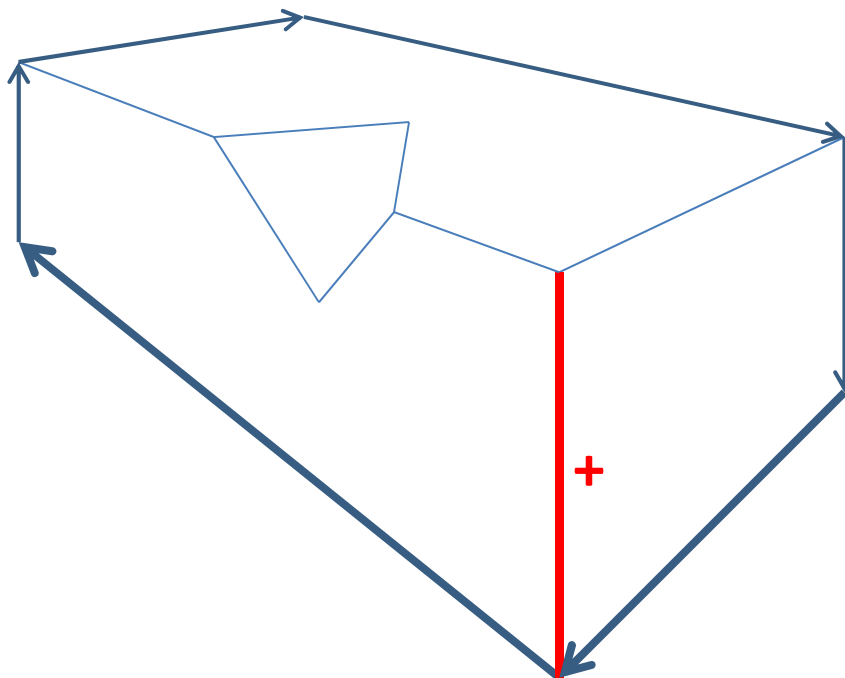
# Solution



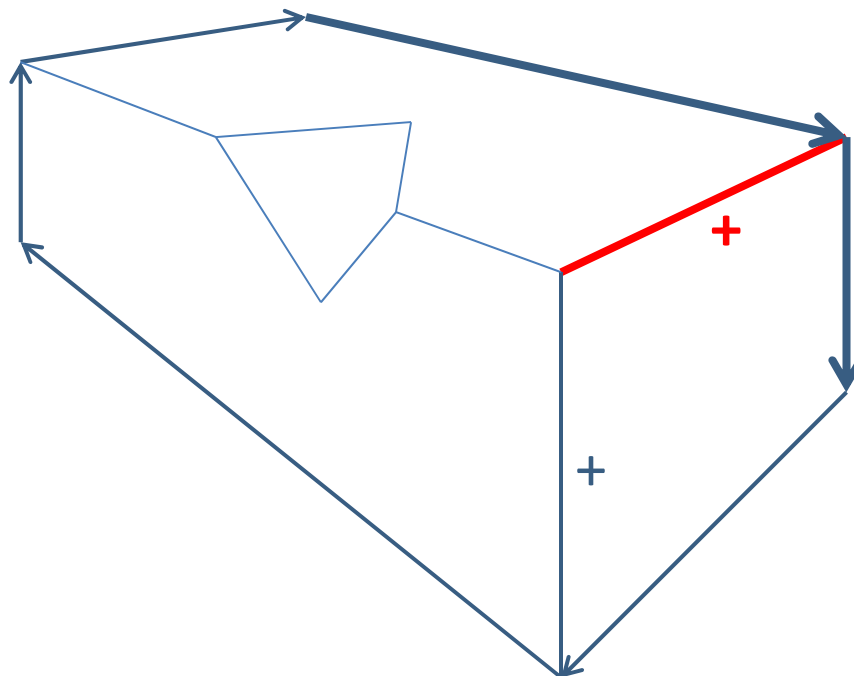
# Solution



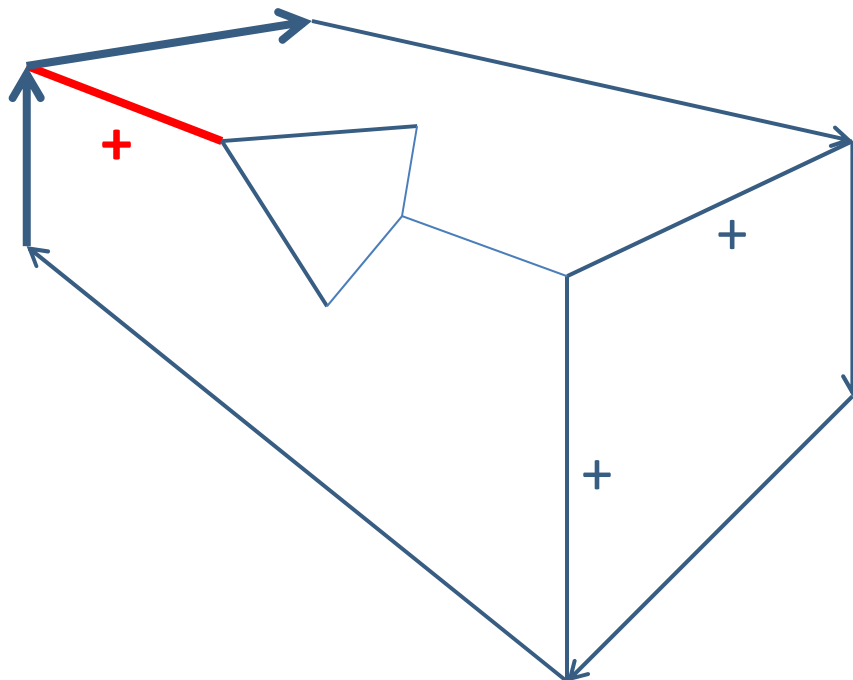
# Solution



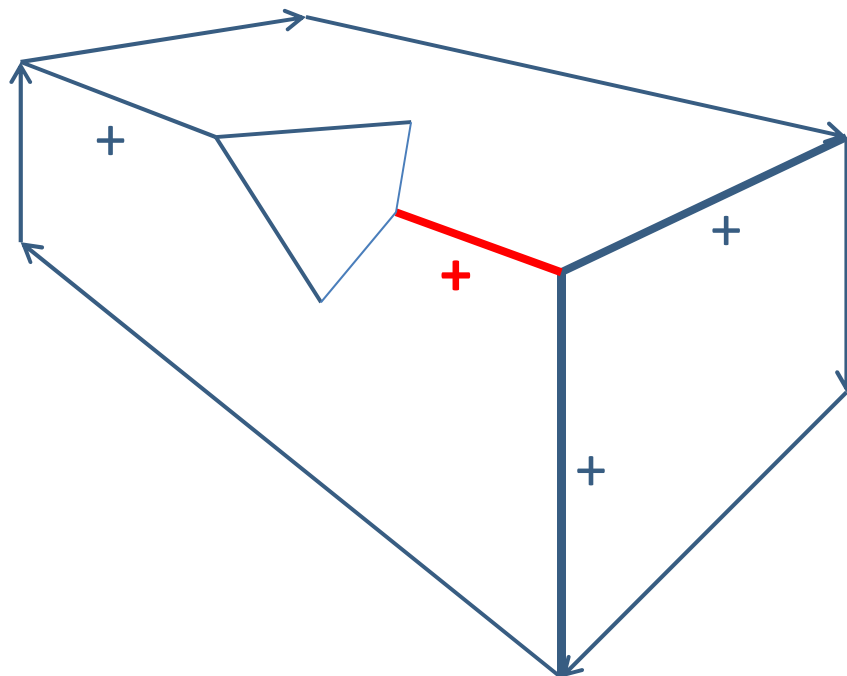
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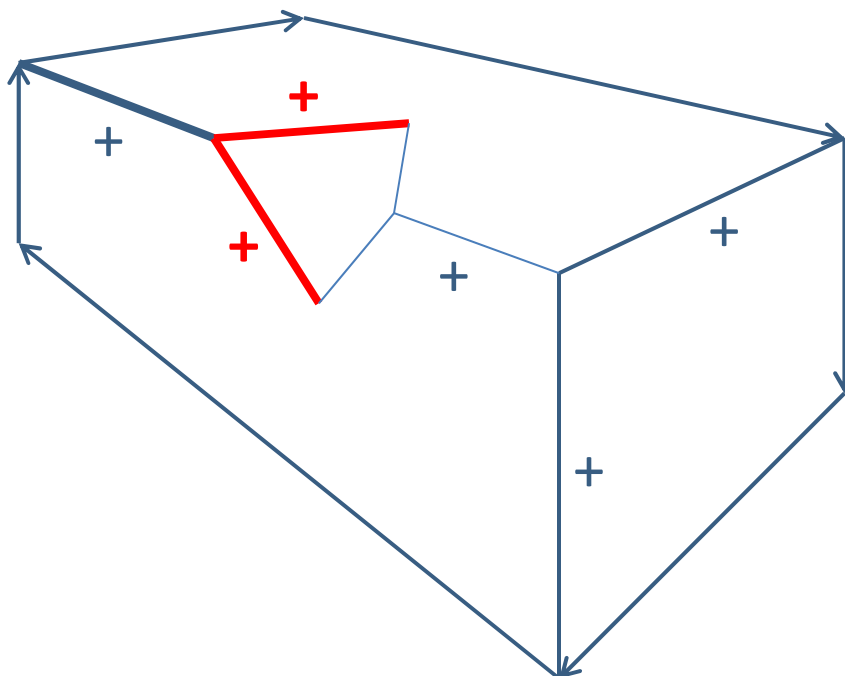
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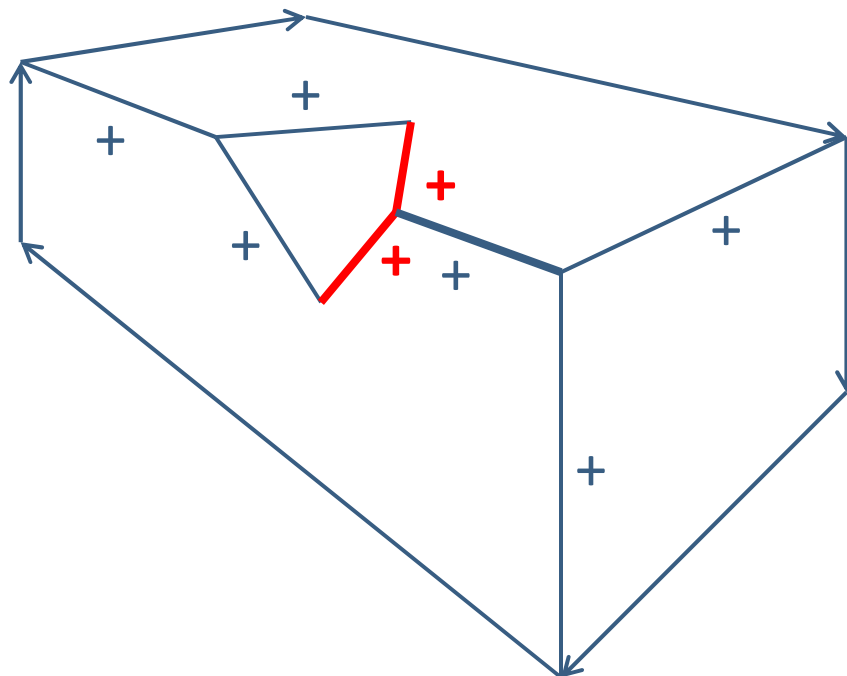
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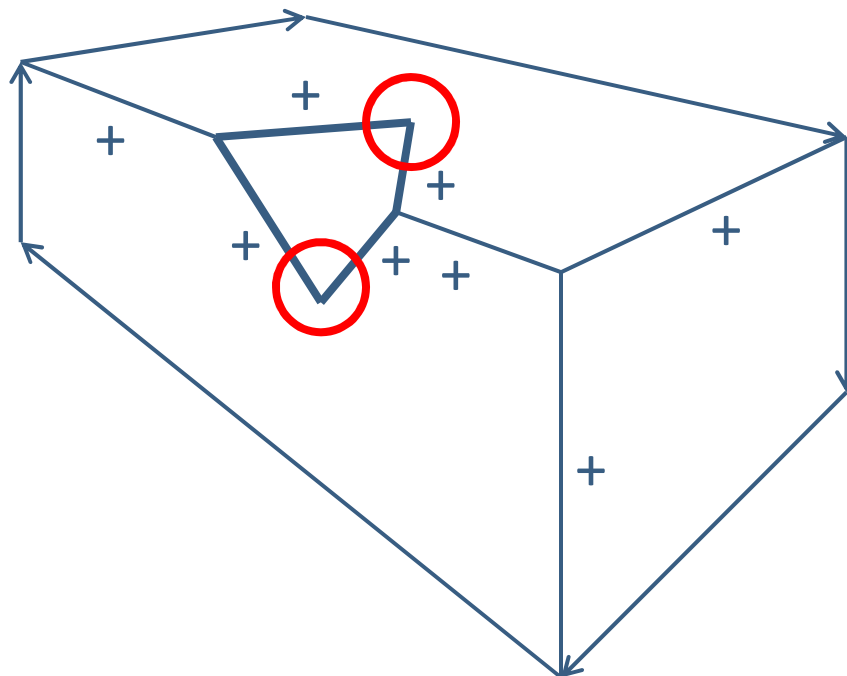
# Solution



# Solution



# Solution

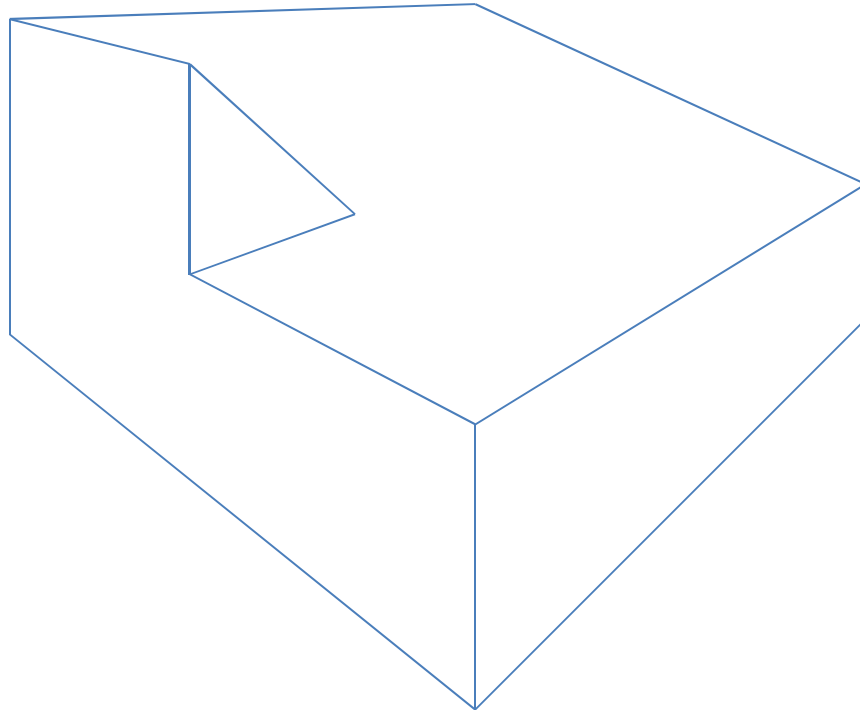


Constraint Processing II & Waltz: Waltz I

# PROBLEM

# Problem

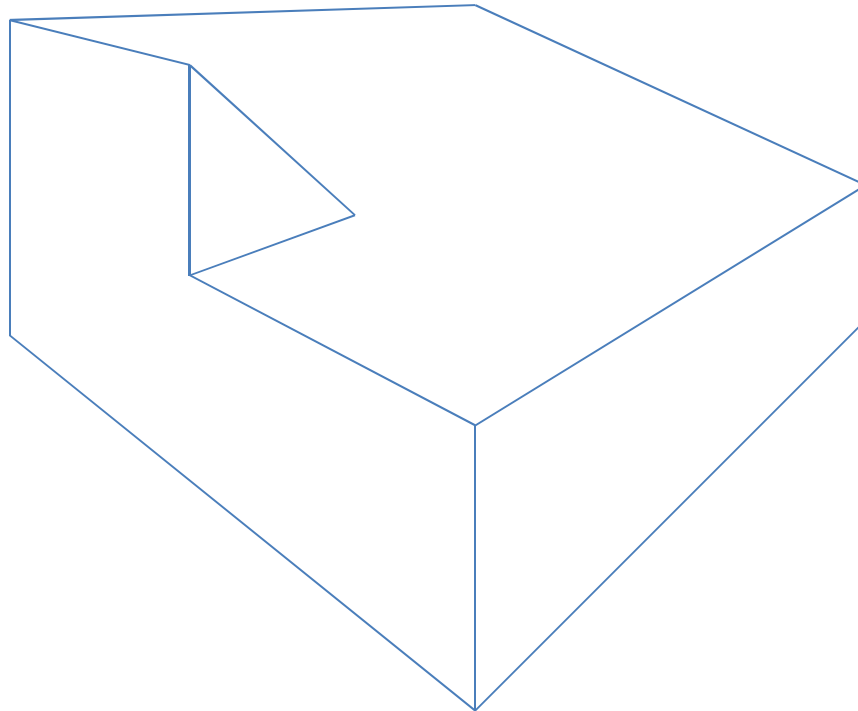
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Constraint Processing II & Waltz: Waltz I

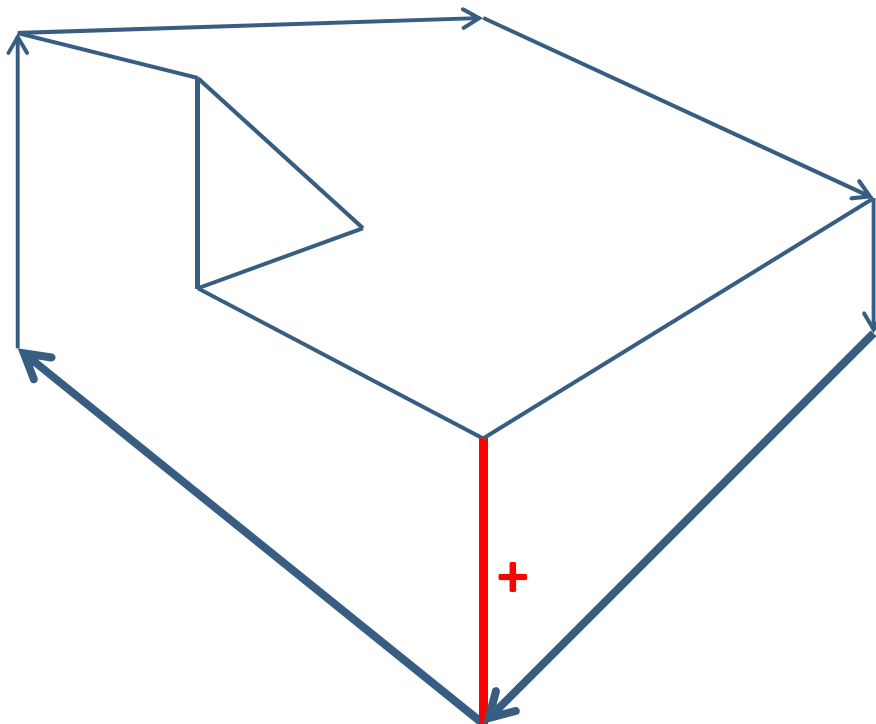
**SOLUTION**

# Solution

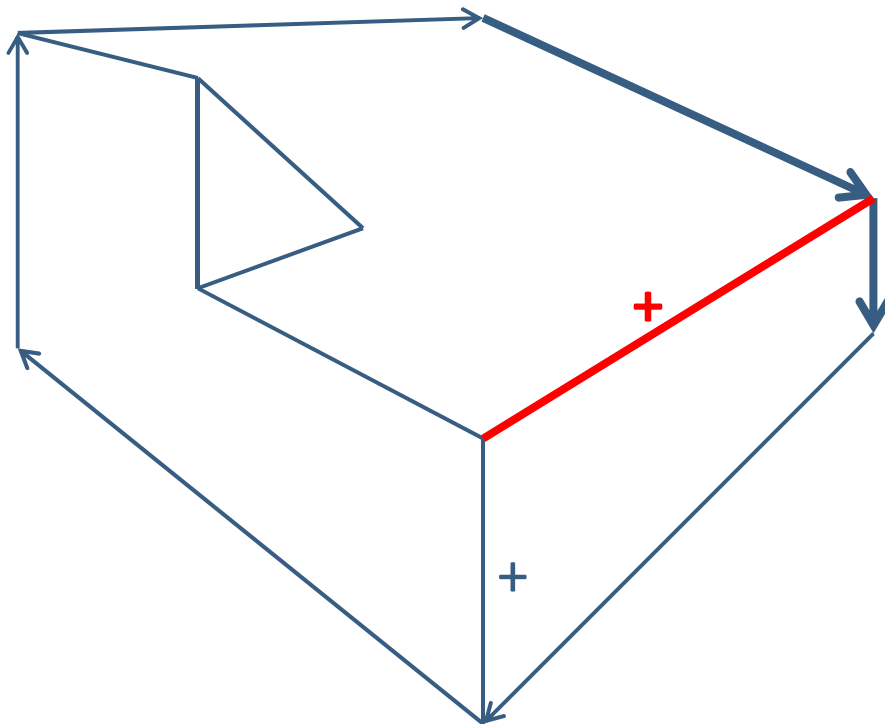




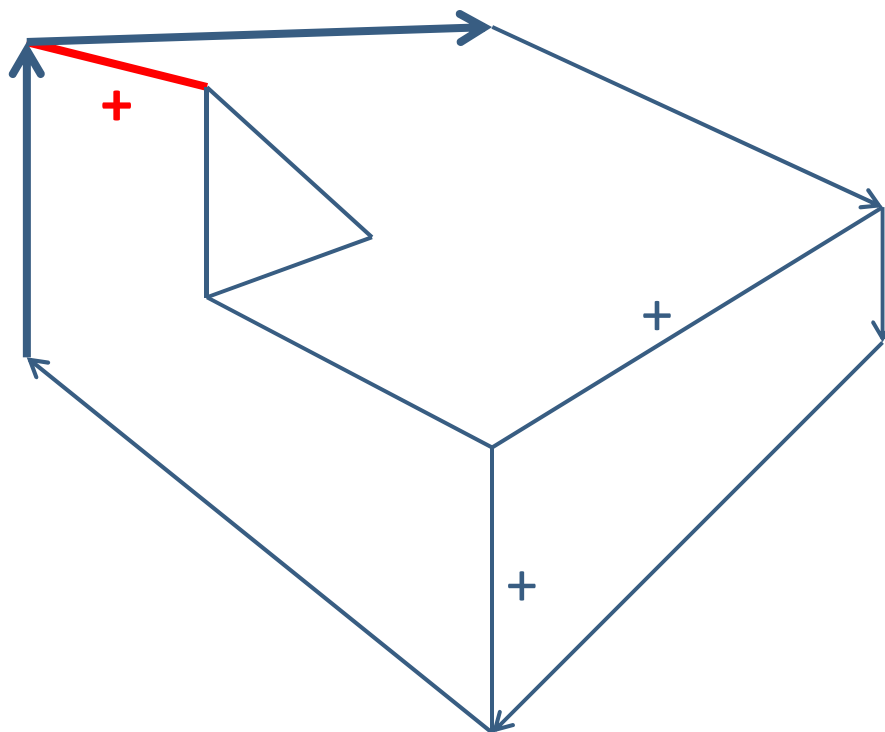
# Solution



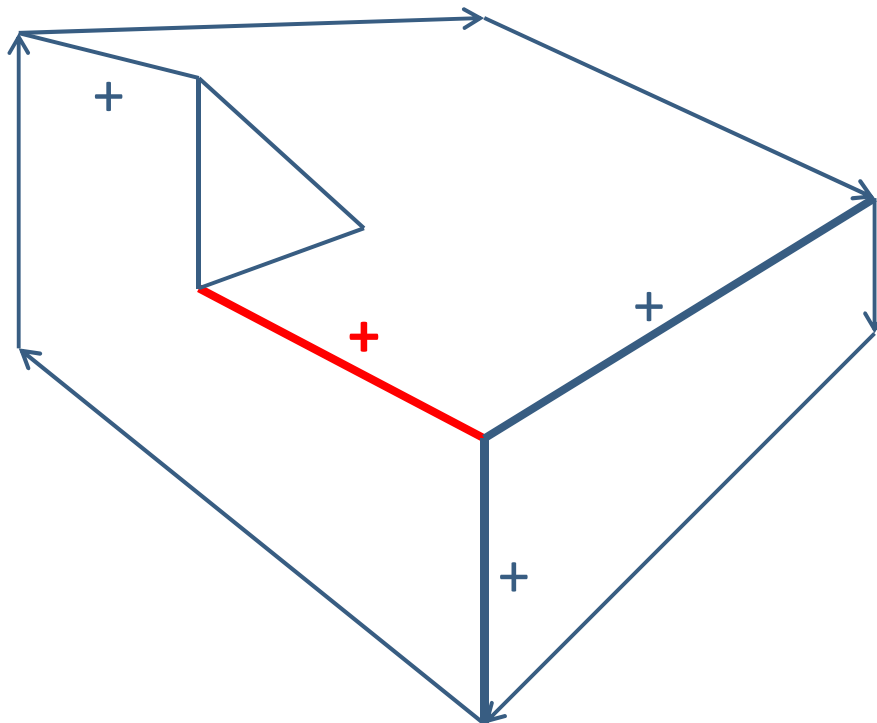
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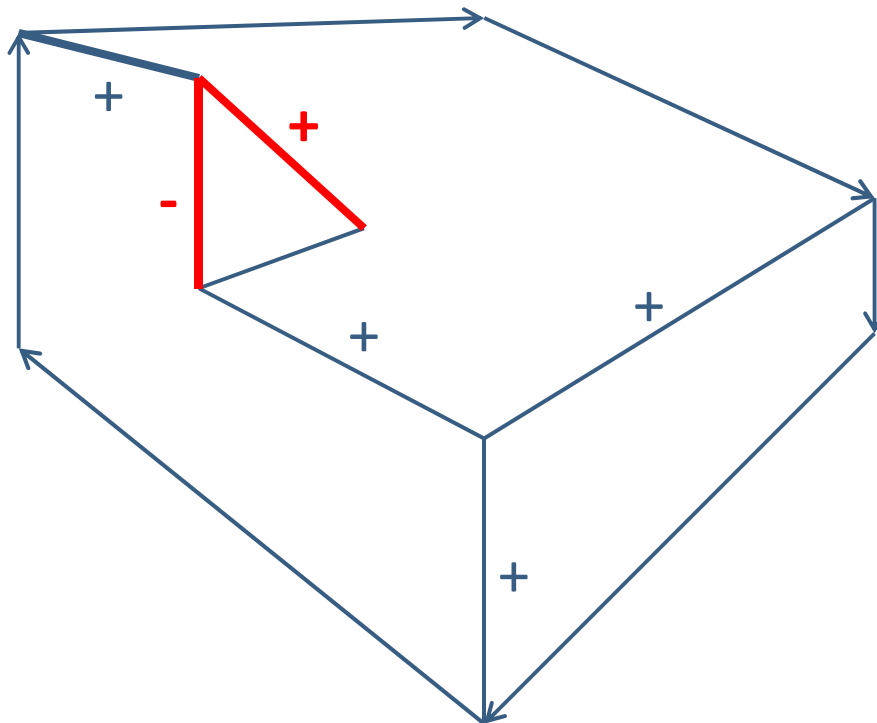
# Solution



# Solution



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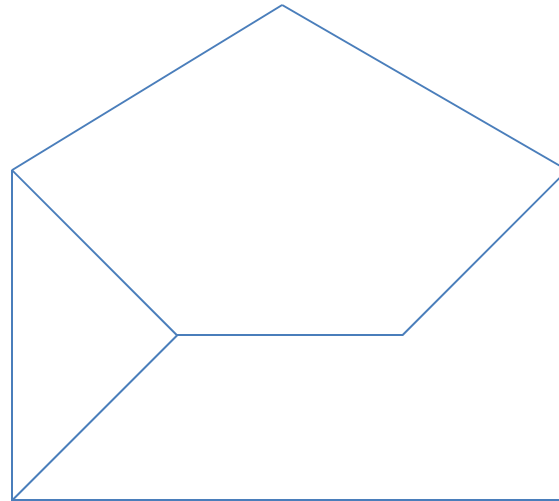


Constraint Processing II & Waltz: Waltz I

# PROBLEM

# Problem

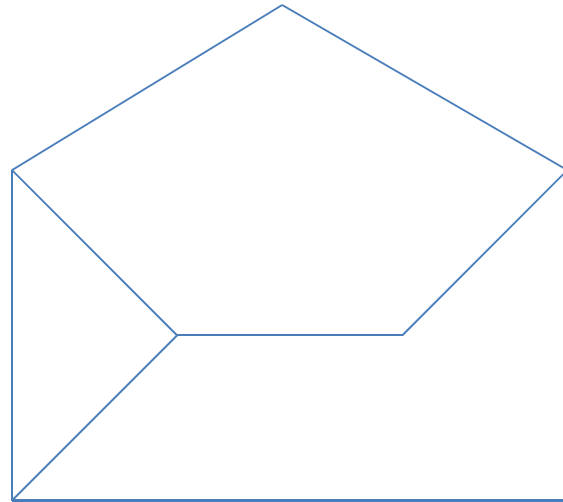
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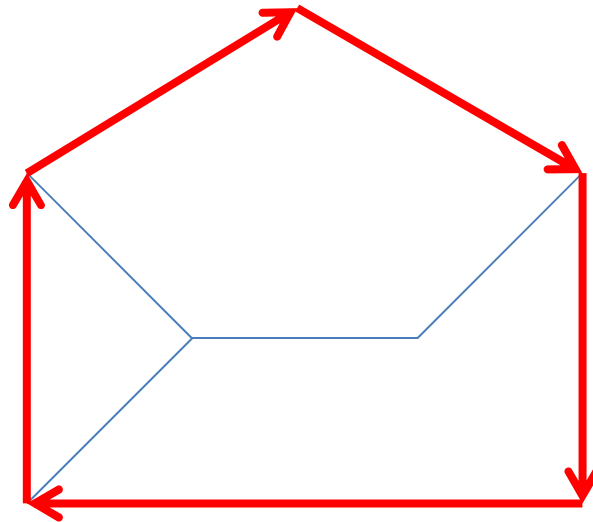
Constraint Processing II & Waltz: Waltz I

**SOLUTION**

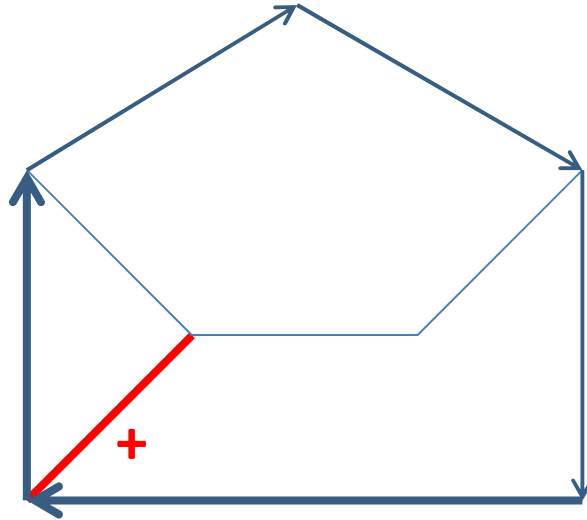
# Solution



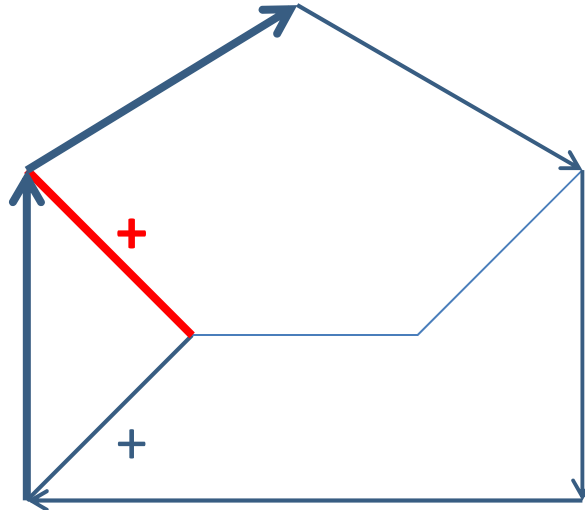
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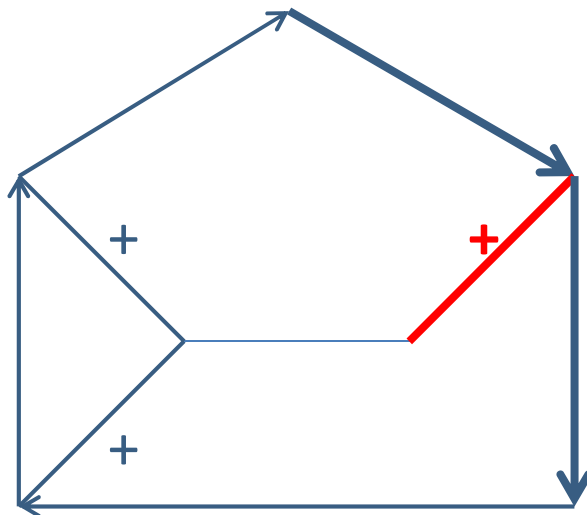
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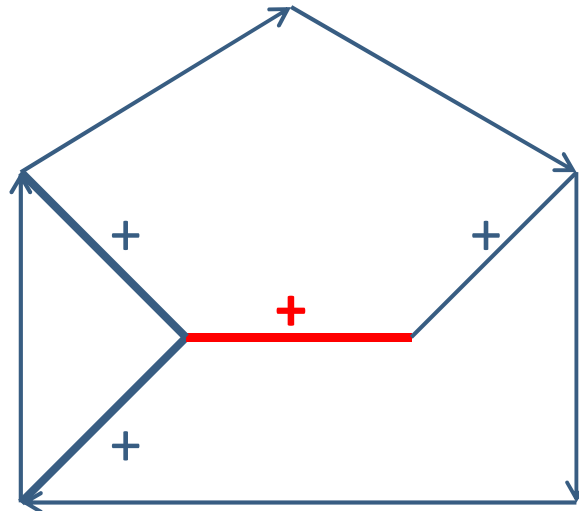
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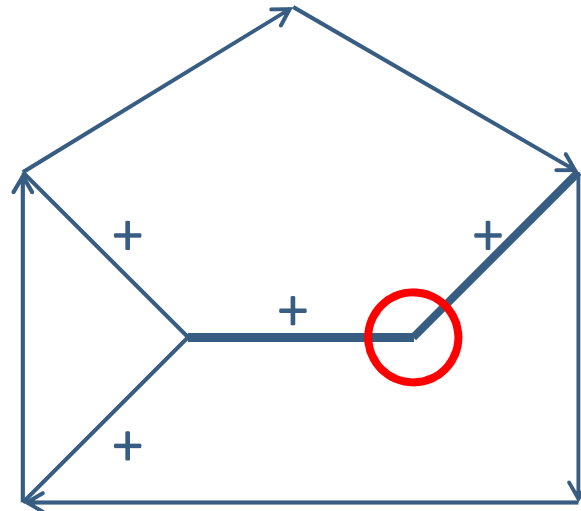
# Solution



# Solution



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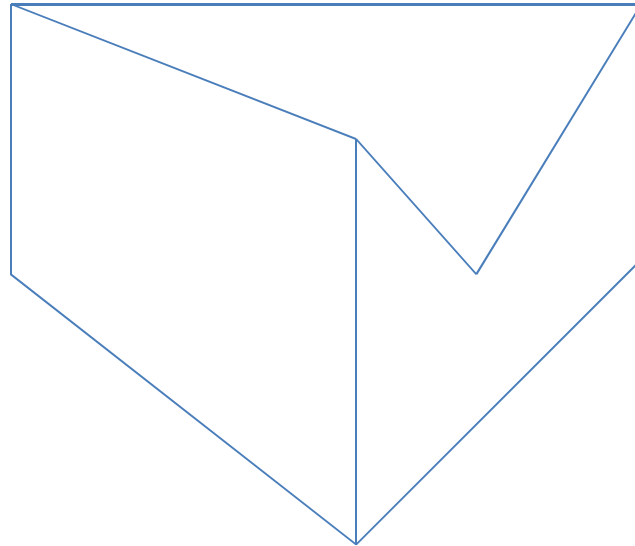


Constraint Processing II & Waltz: Waltz I

# PROBLEM

# Problem

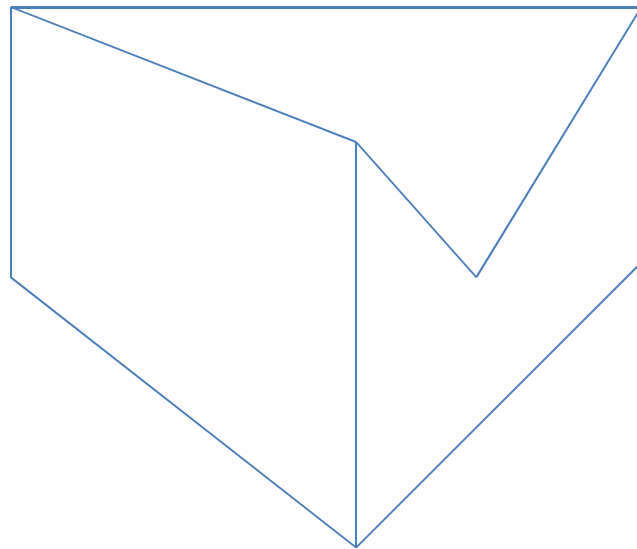
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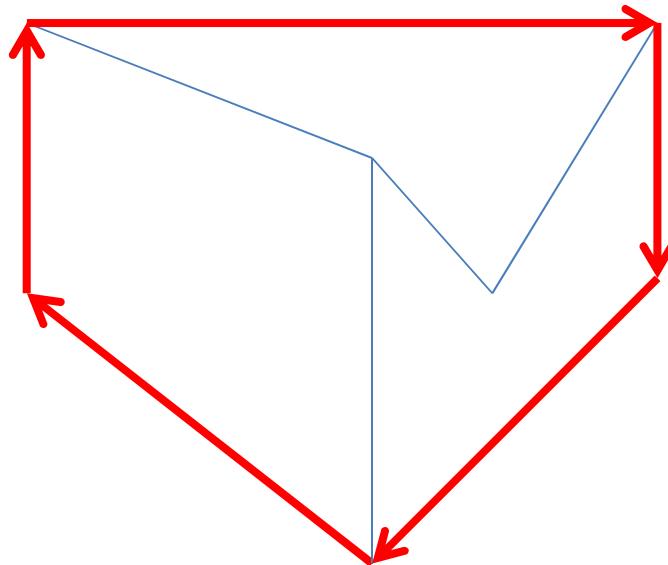
Constraint Processing II & Waltz: Waltz I

**SOLUTION**

# Solution

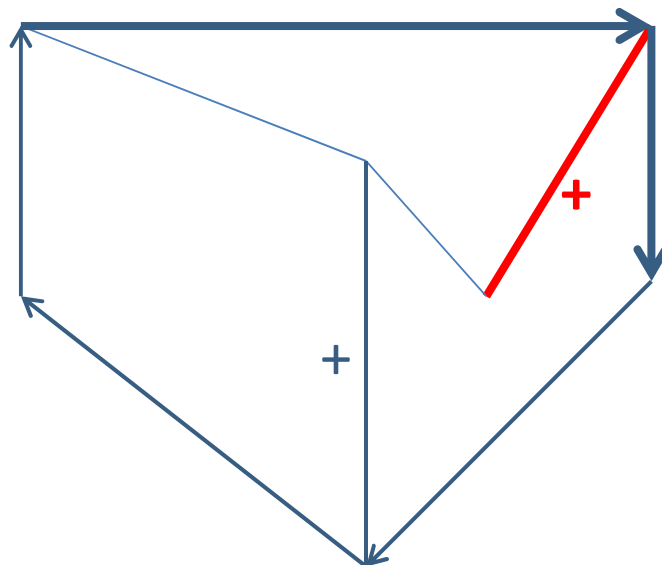


# Solution

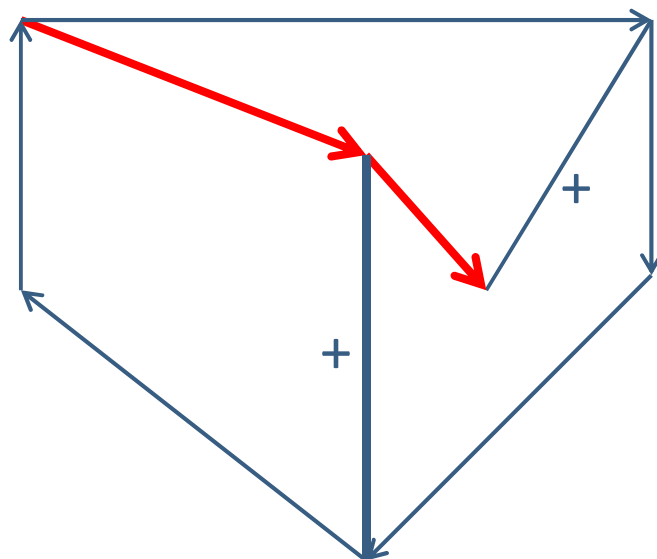




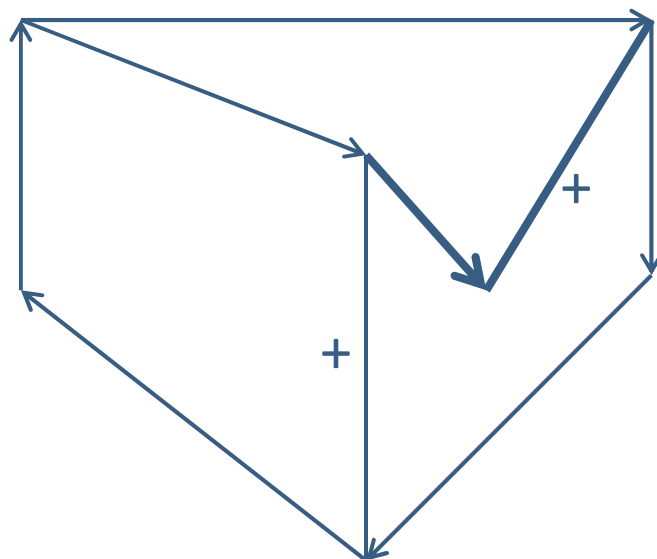
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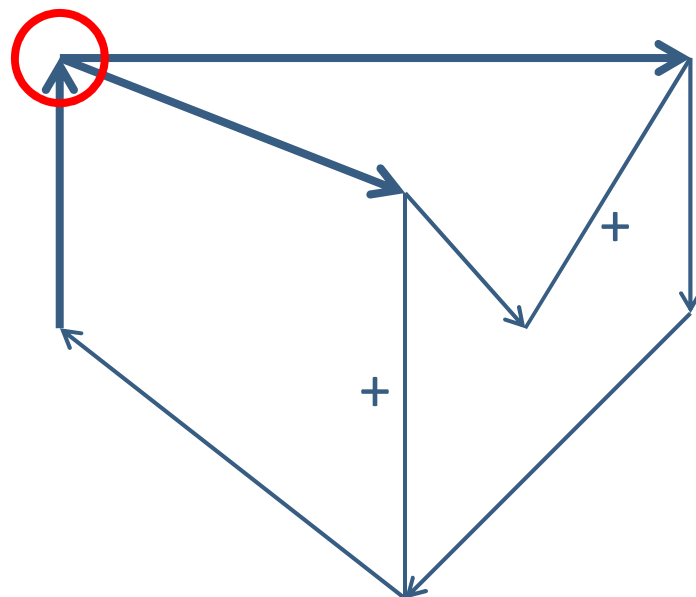
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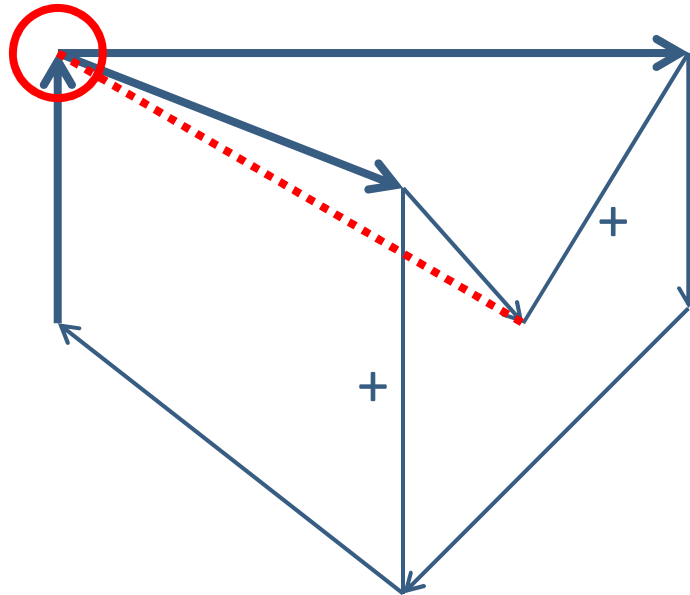


# Solution



# Solution

Line Drawing NOT allowed: 3-faced vertices!!

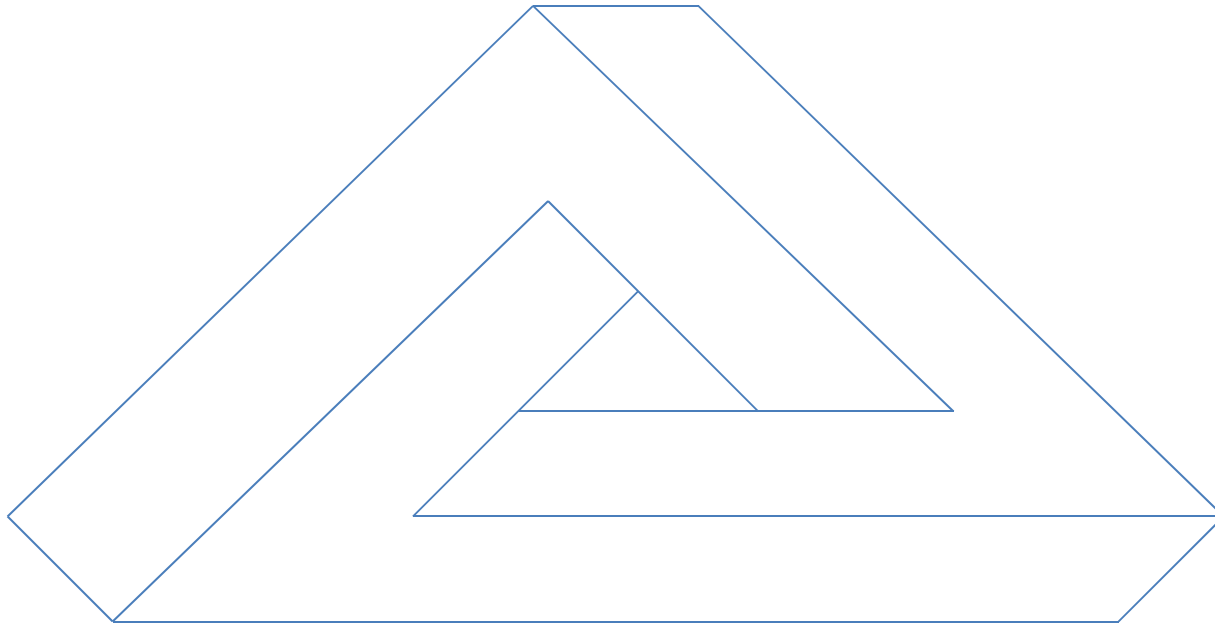


Constraint Processing II & Waltz: Waltz I

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# Problem

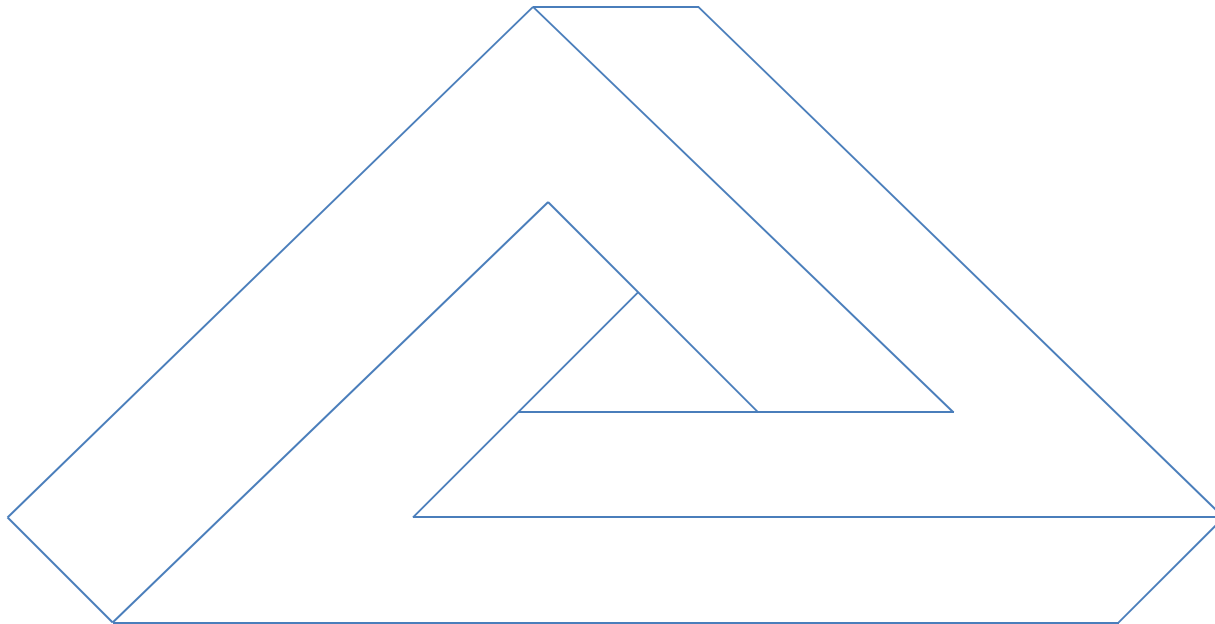
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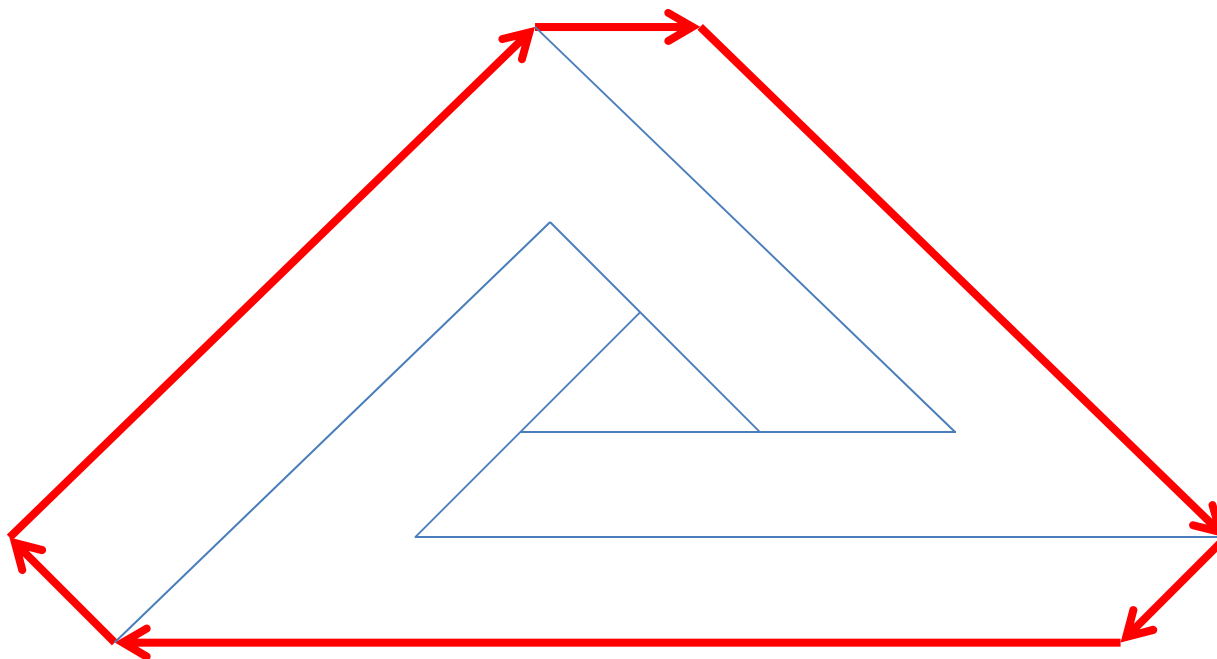
Constraint Processing II & Waltz: Waltz I

**SOLUTION**

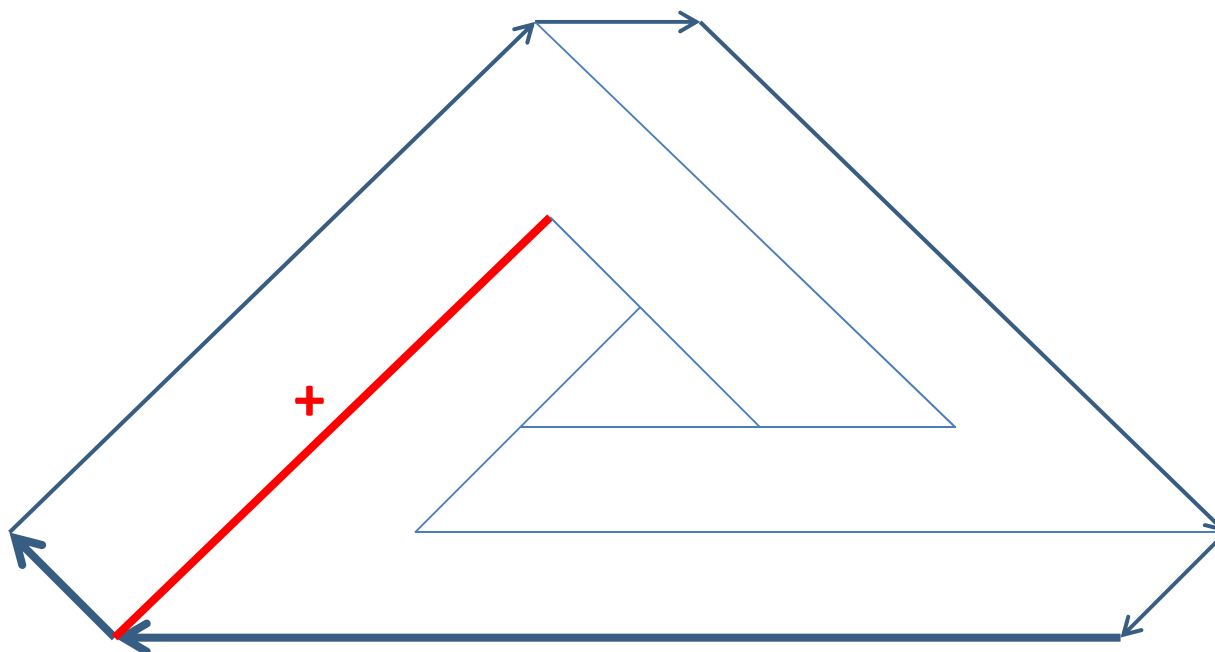
# Solution



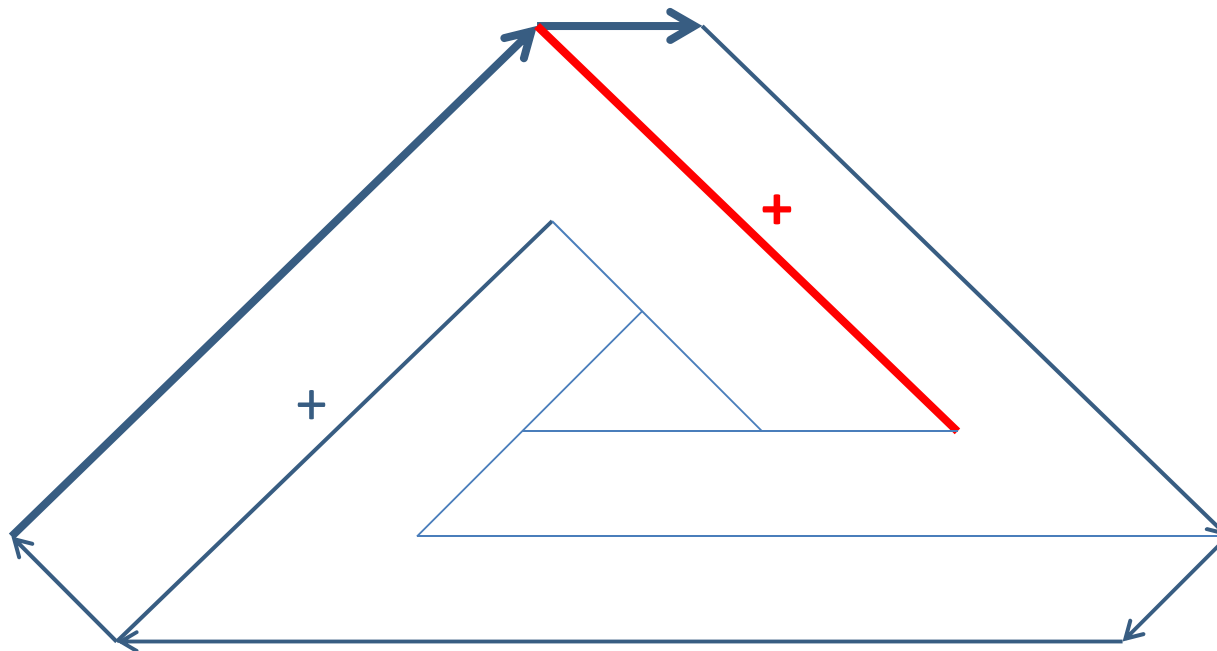
# Solution



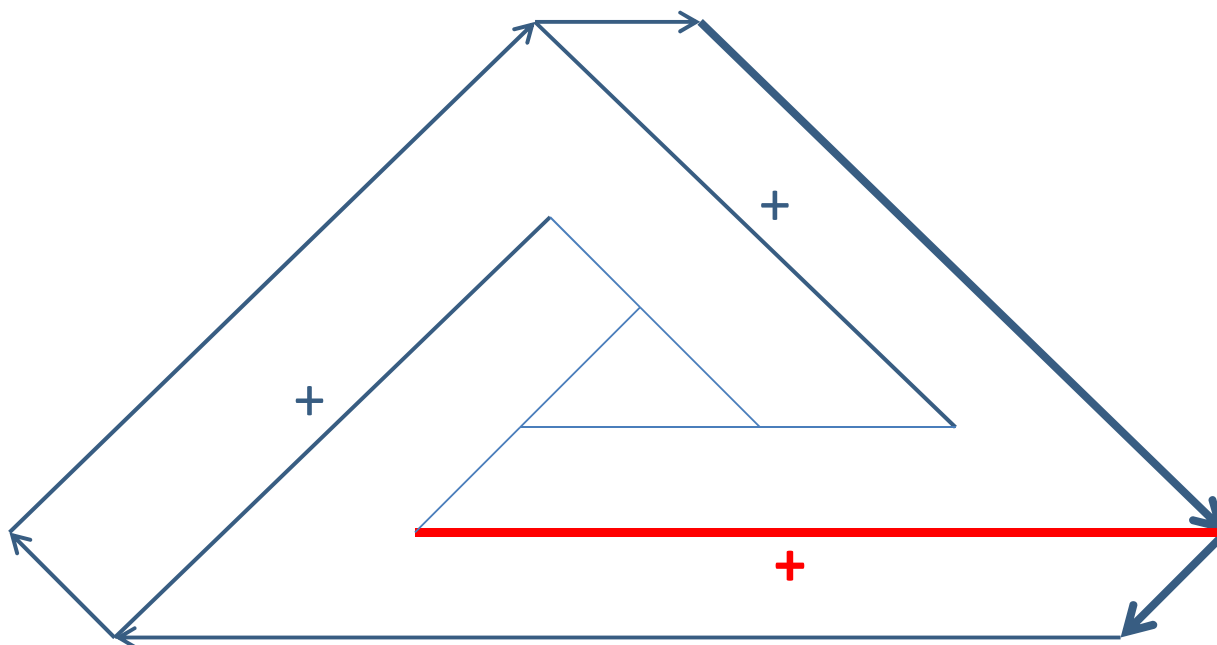
# Solution



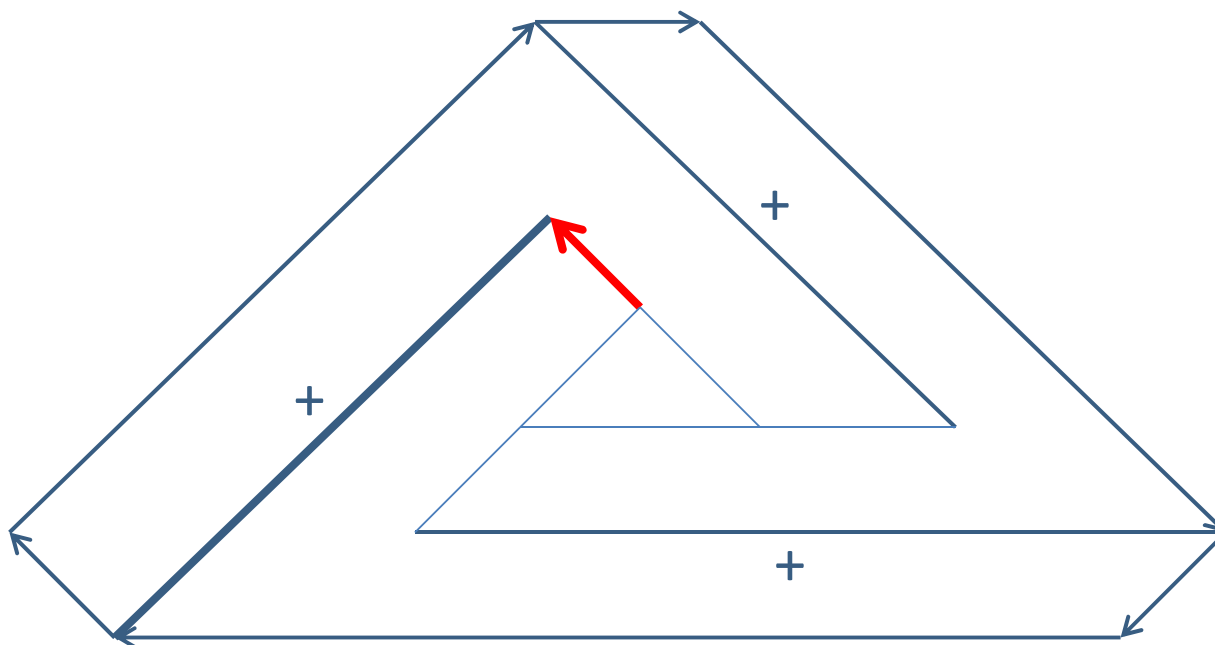
# Solution



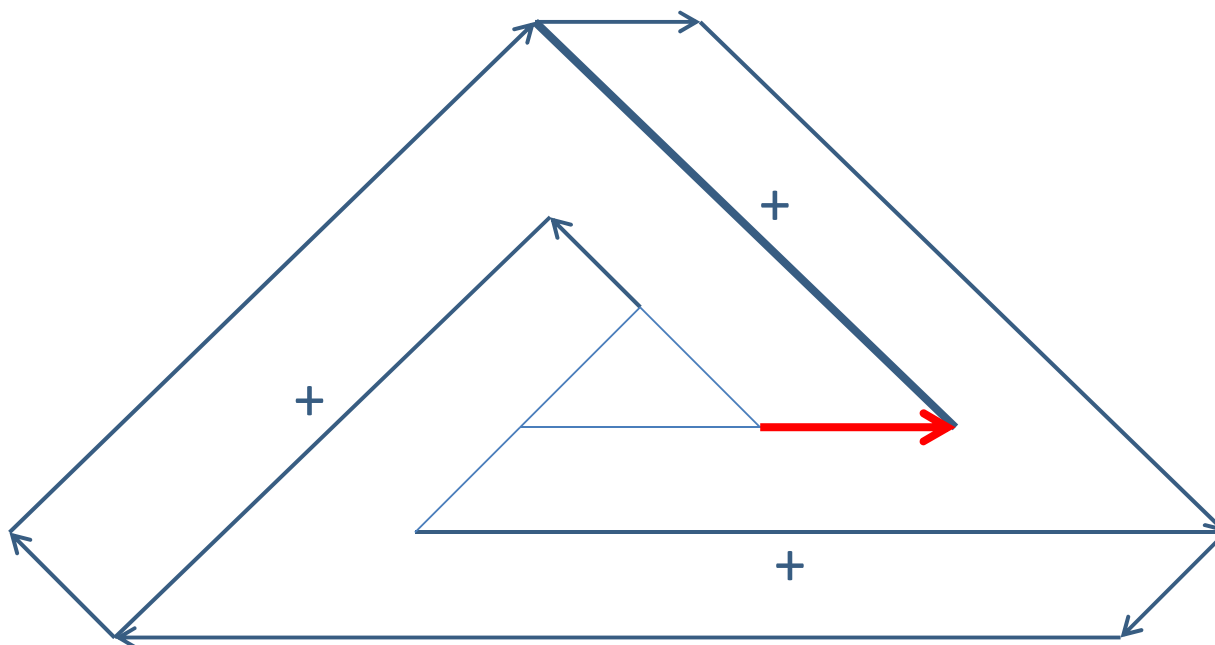
# Solution



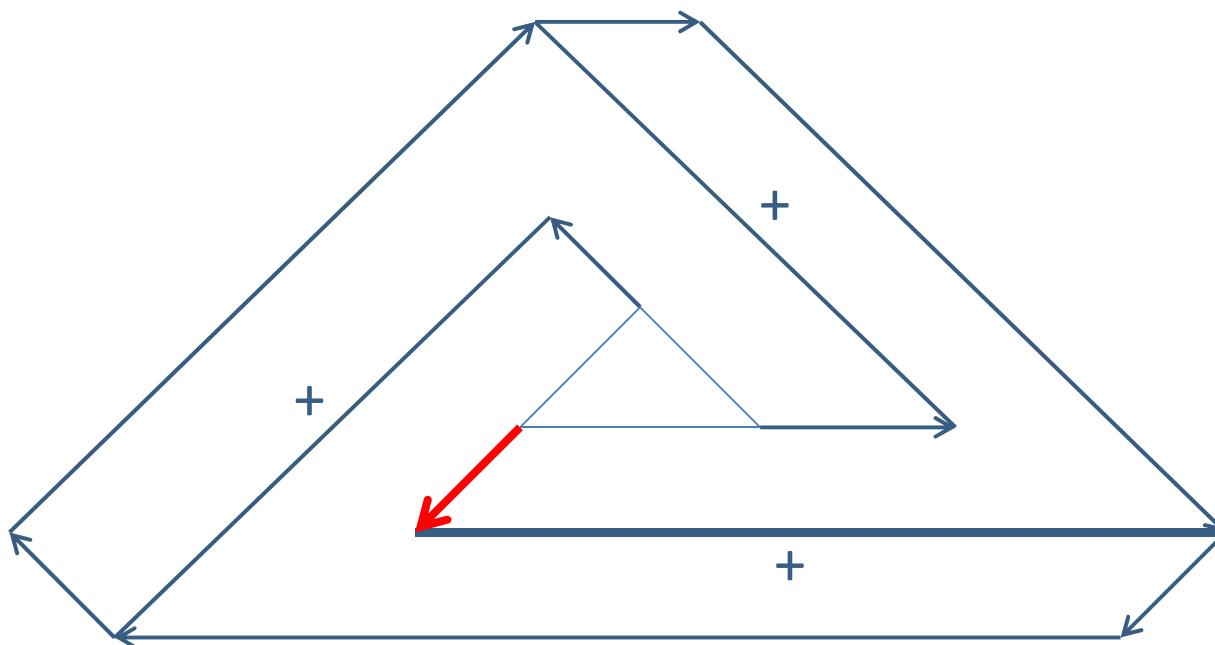
# Solution



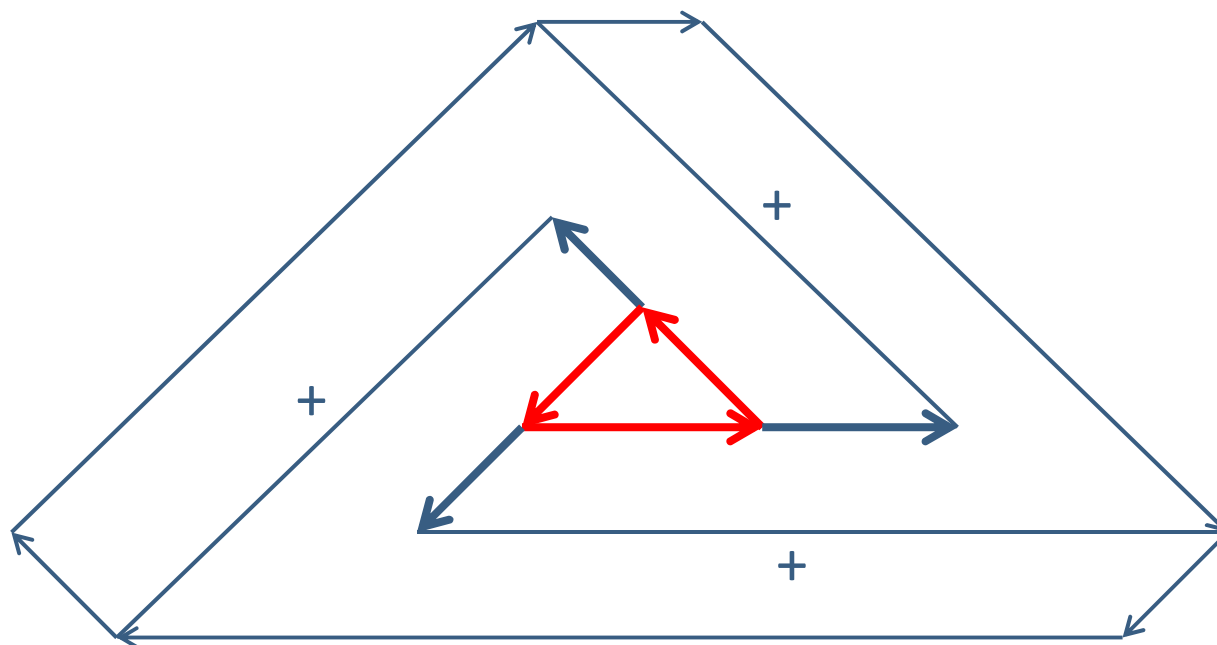
# Solution



# Solution

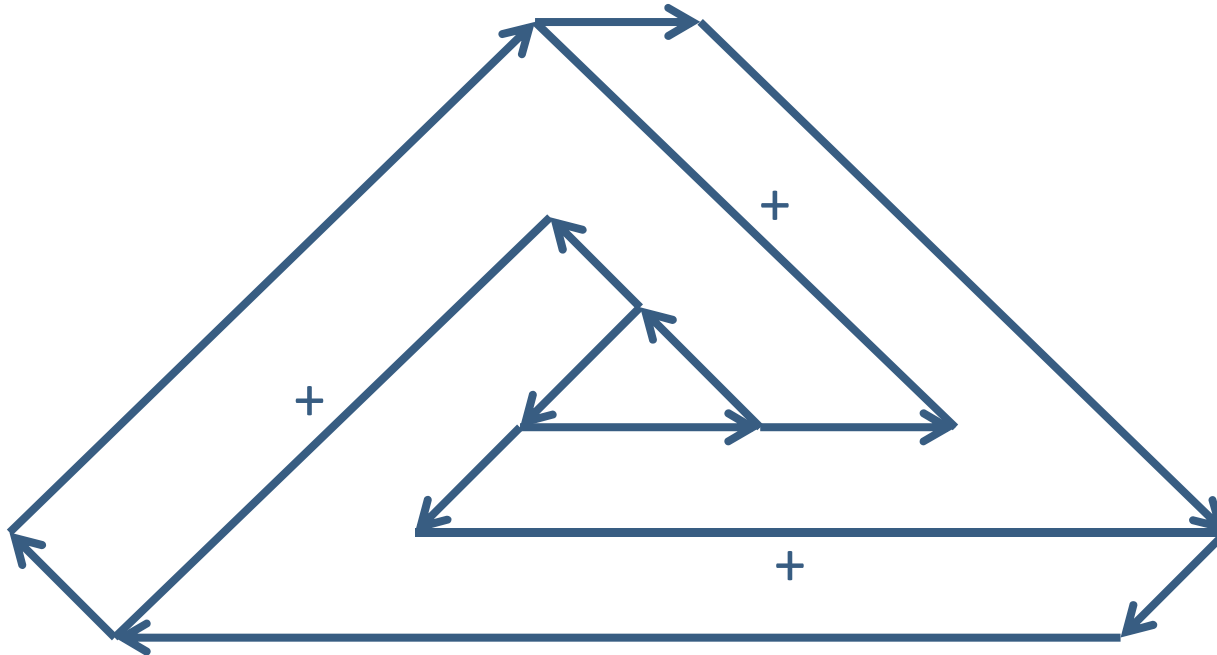


# Solution



# Solution

Drawing is locally correct, but is globally impossible.  
Waltz procedure is local, thus, cannot detect this!



# Exercises: Artificial Intelligence

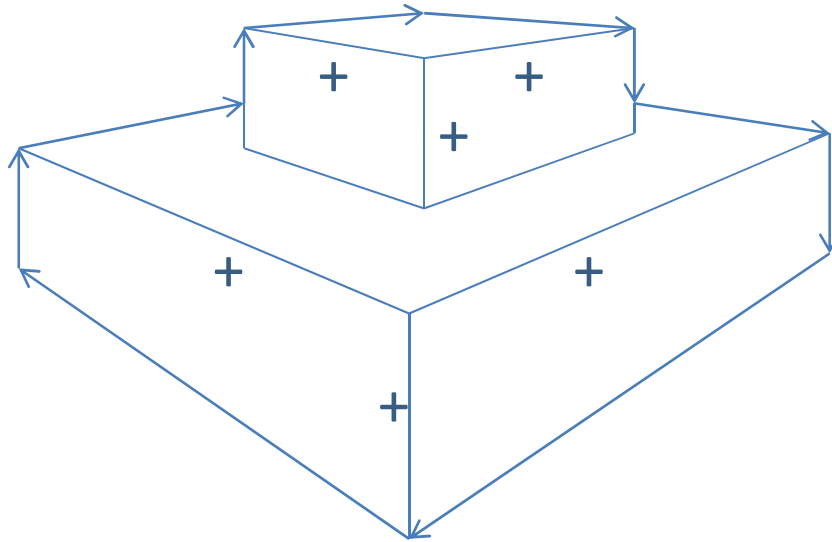
Constraint Processing II & Waltz:  
Waltz II

Constraint Processing II & Waltz: Waltz II

# **PROBLEM**

# Problem

- Finish the labeling of the following figure.
- Give all solutions:

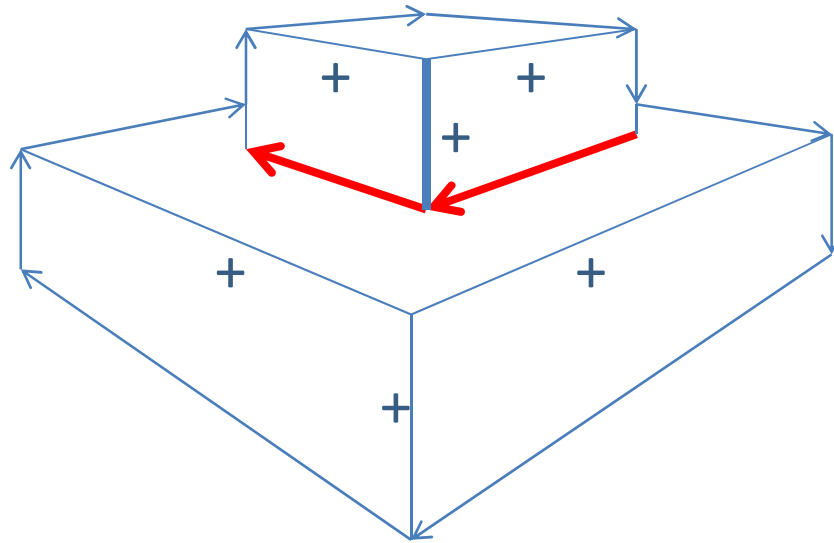


Constraint Processing II & Waltz: Waltz II

**SOLUTION**

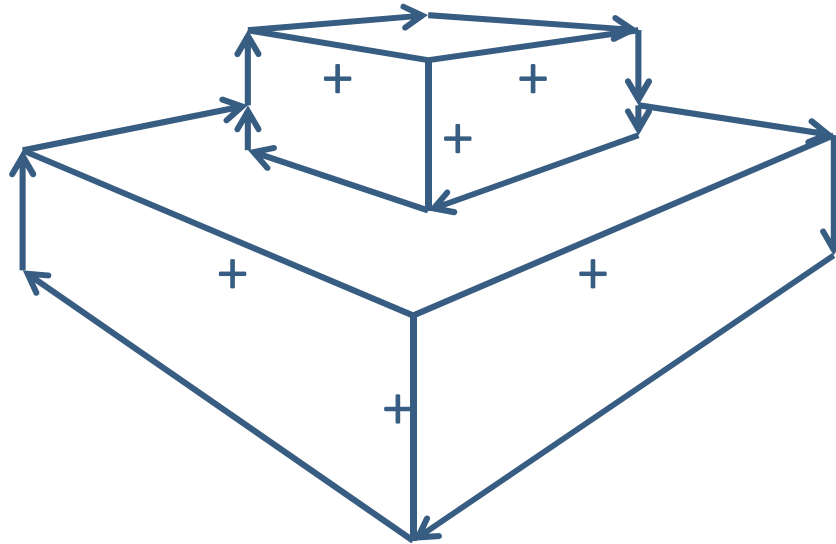
# Solution

- Solution 1: Floating cube



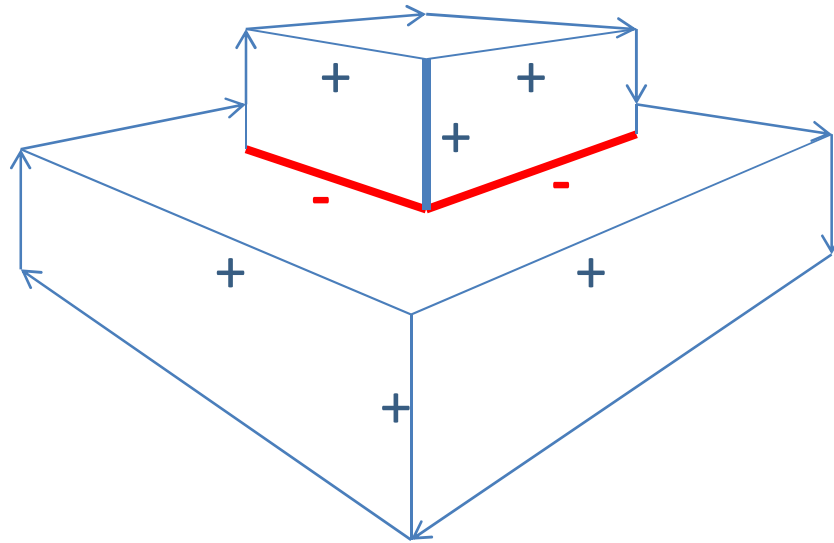
# Solution

- Solution 1: Floating cube



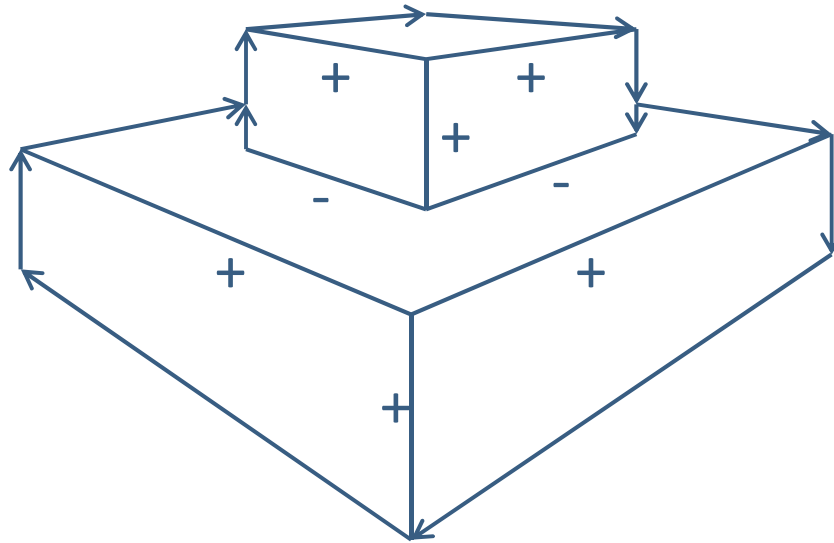
# Solution

- Solution 2: Sitting cube



# Solution

- Solution 2: Sitting cube



# Exercises: Artificial Intelligence

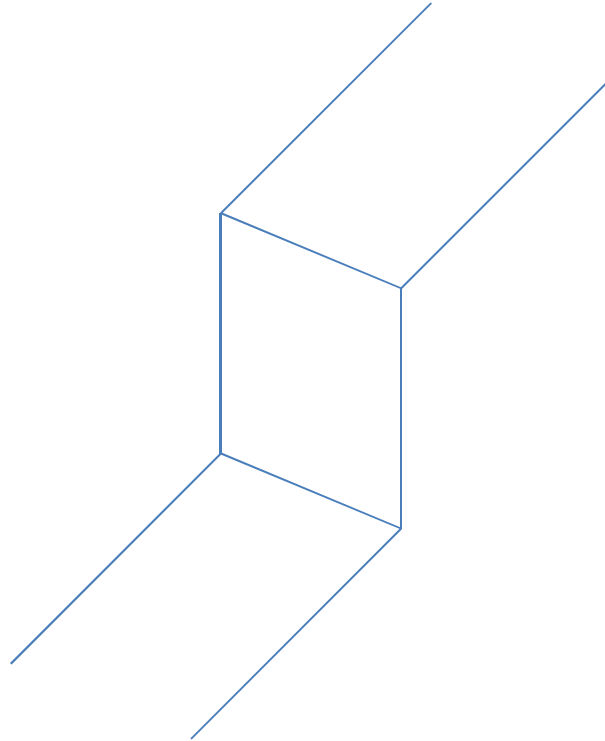
Constraint Processing II & Waltz:  
Waltz III

Constraint Processing II & Waltz: Waltz III

# **PROBLEM**

# Problem

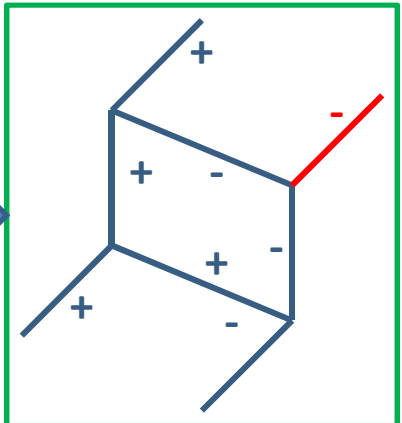
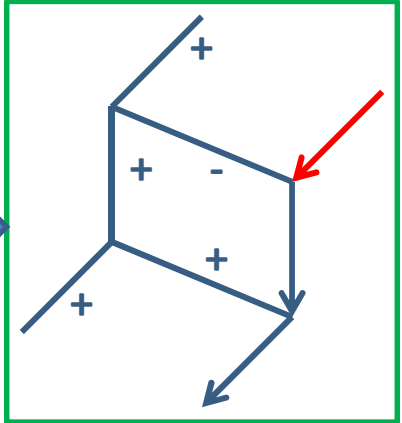
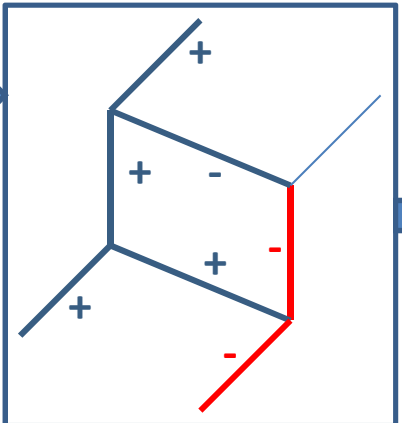
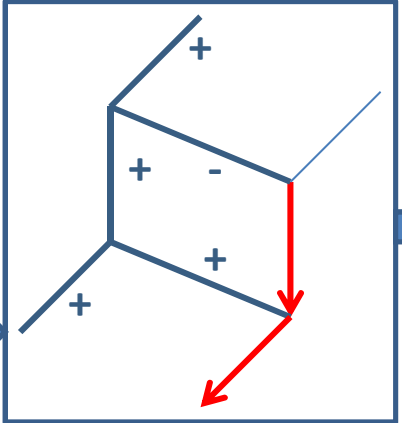
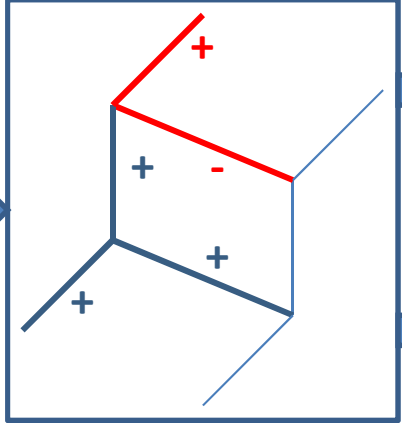
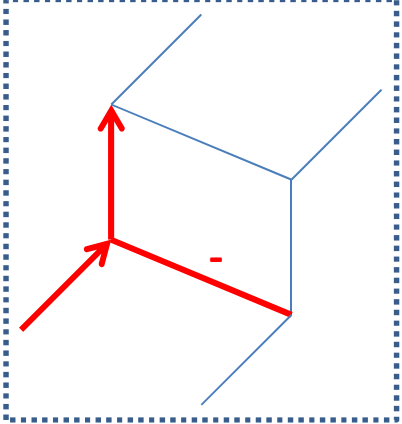
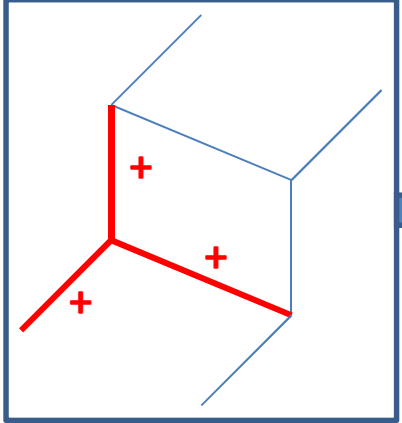
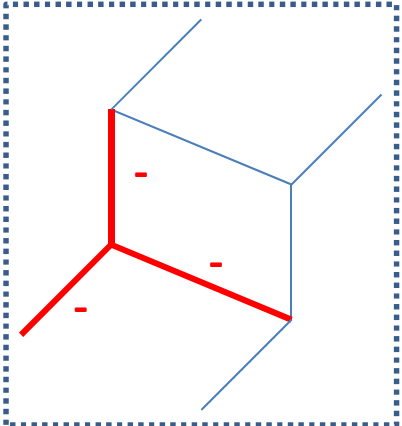
- Labeling the following figure fragment.
- Give all solutions:



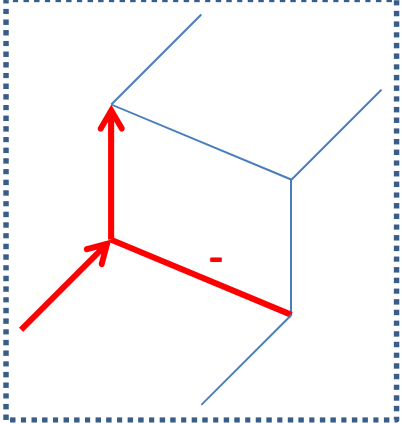
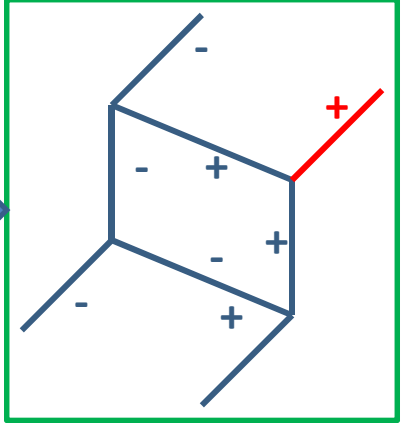
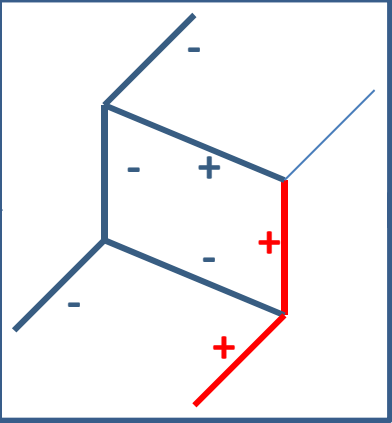
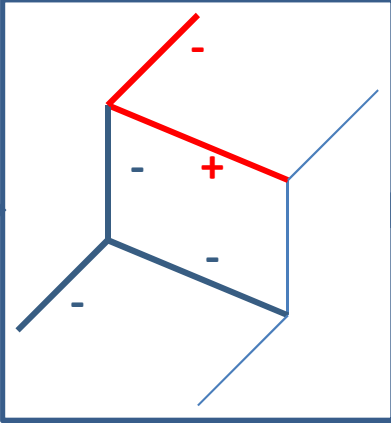
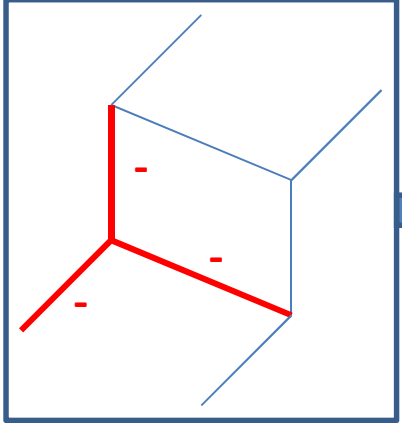
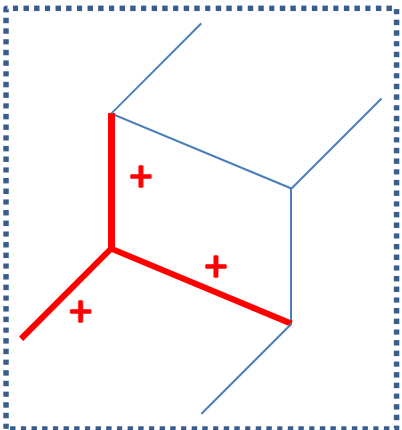
Constraint Processing II & Waltz: Waltz III

**SOLUTION**

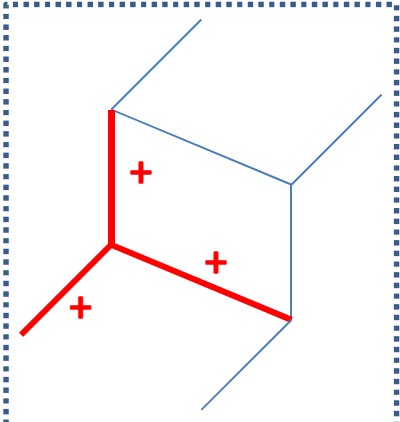
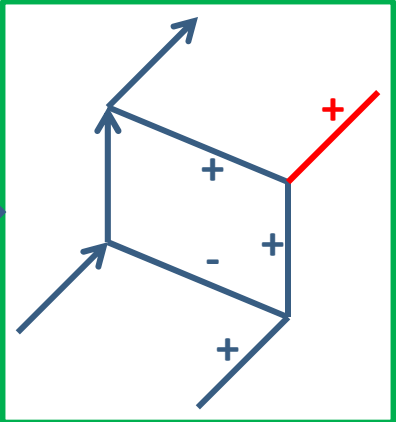
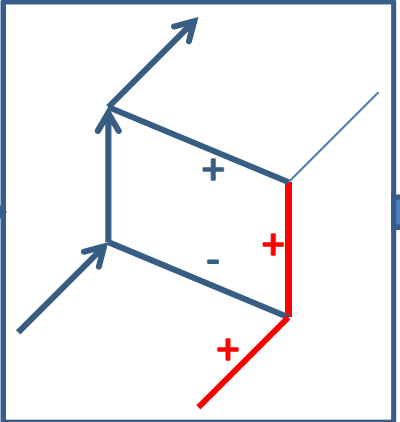
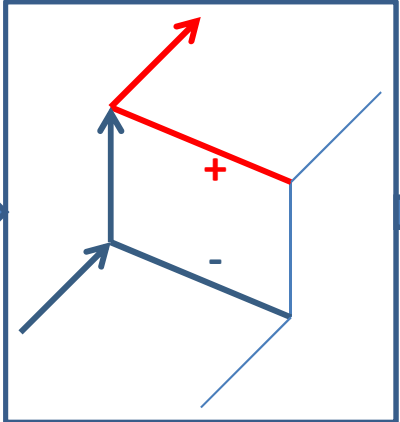
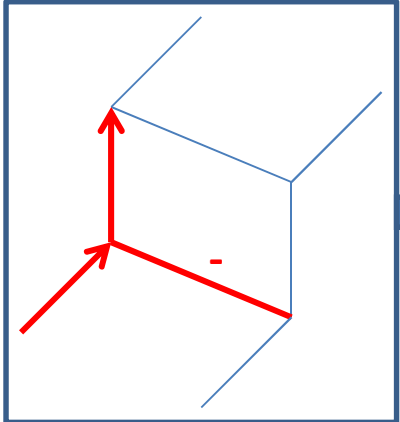
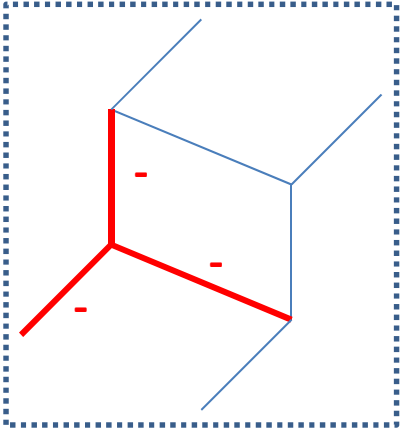
# Solution



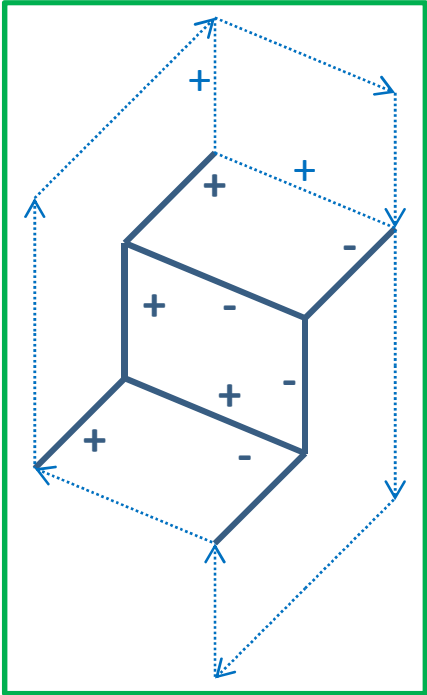
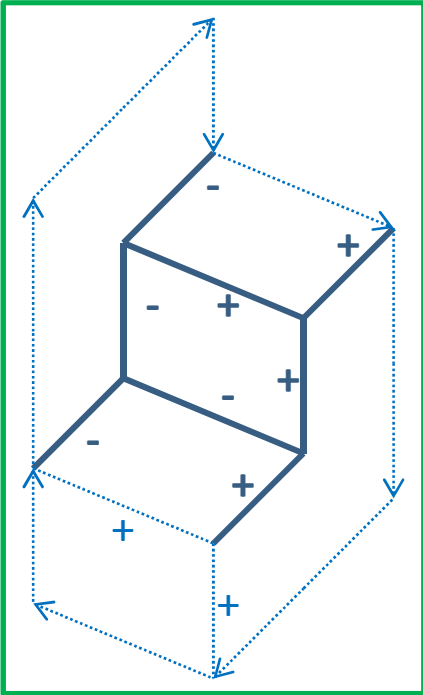
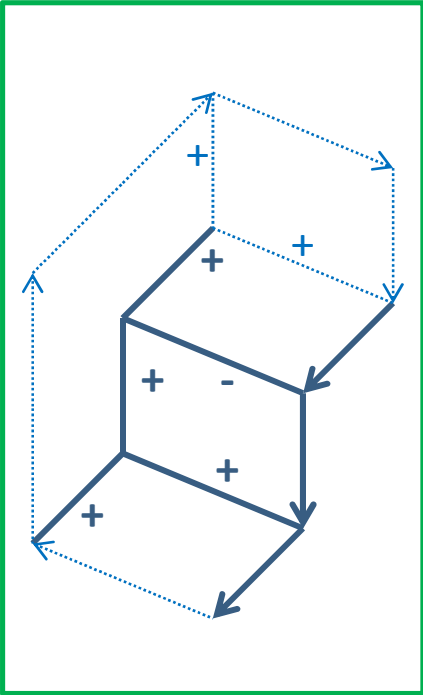
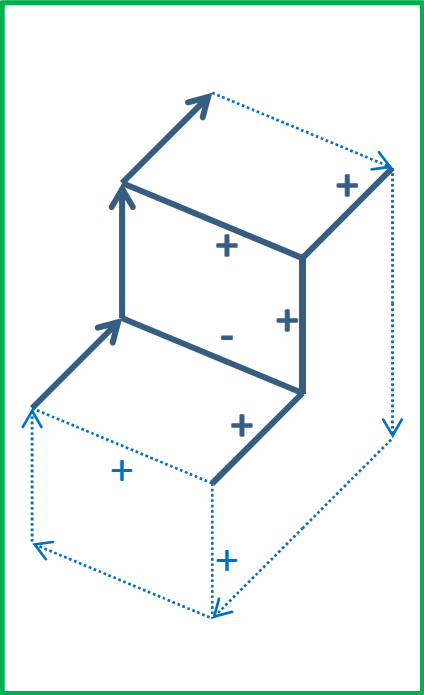
# Solution



# Solution



# Solution



# Exercises: Artificial Intelligence

Constraint Processing II & Waltz:  
Waltz IV

Constraint Processing II & Waltz: Waltz IV

# **PROBLEM**

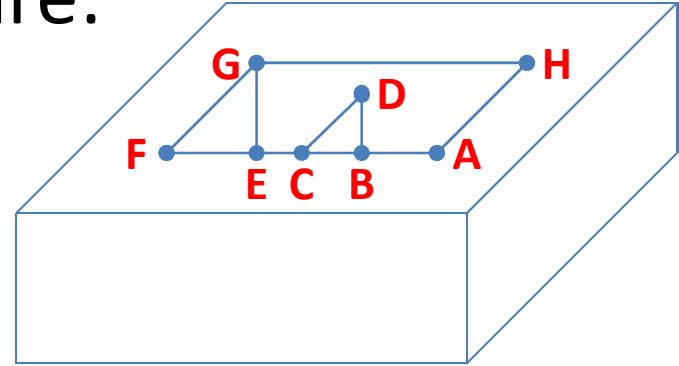
# Problem

- Consider the following figure:

- At most 3 edges
- No shadows/cracks

- Find labelings:

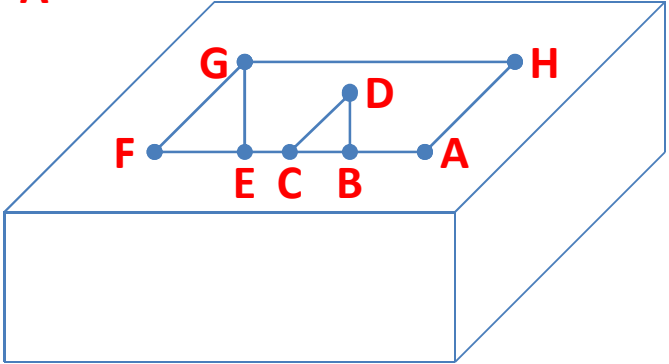
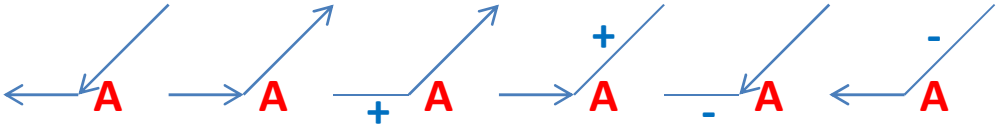
- Write down junction piles for A,B,...,H
- Consider nodes in order of A,B,...,H
  - Return to previous nodes for pruning if possible



Constraint Processing II & Waltz: Waltz IV

**SOLUTION**

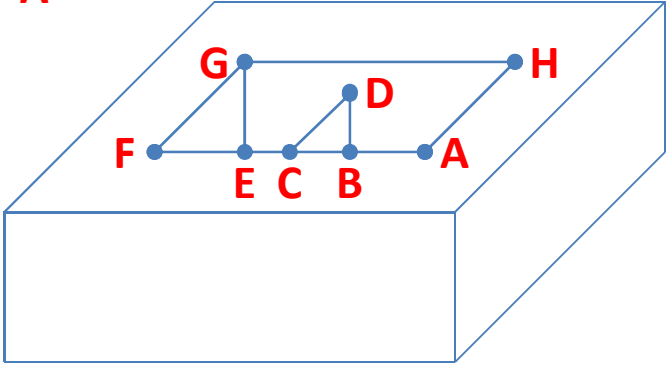
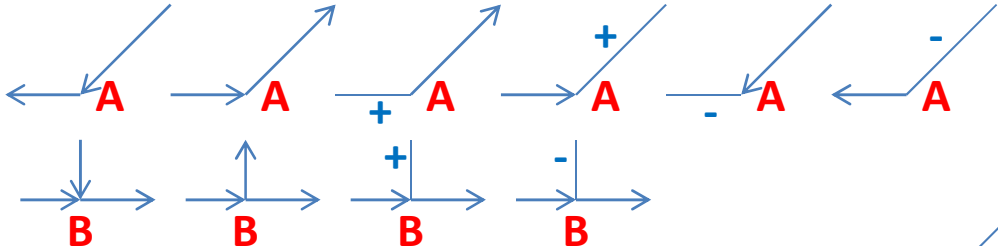
# Solution



# Solution

**A**

**B**

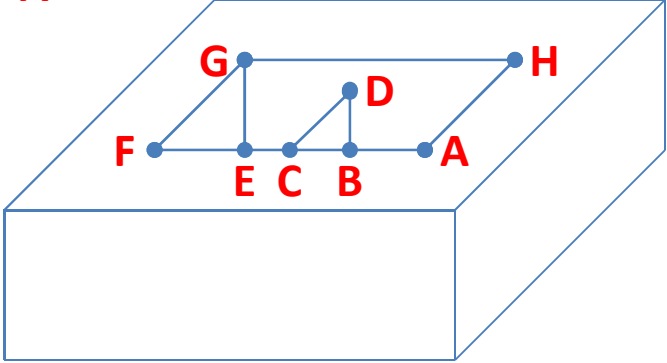
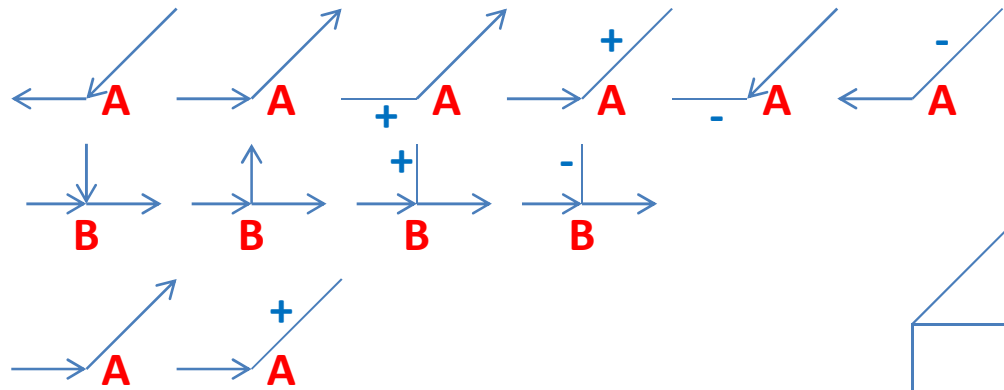


# Solution

A

B

A



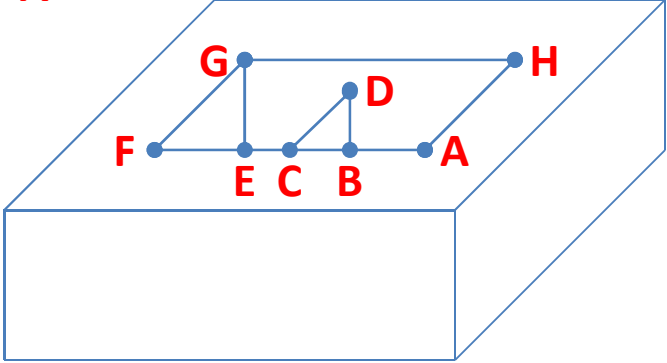
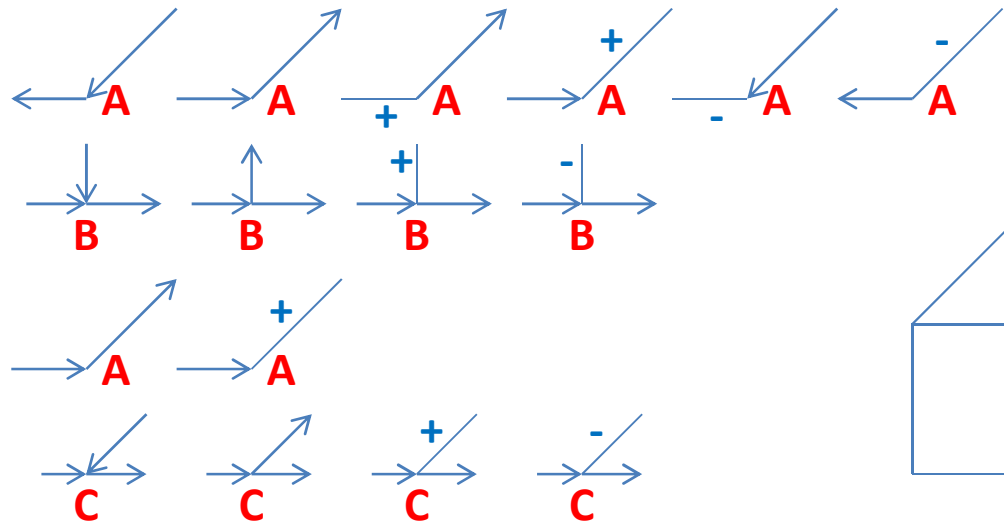
# Solution

A

B

A

C



# Solution

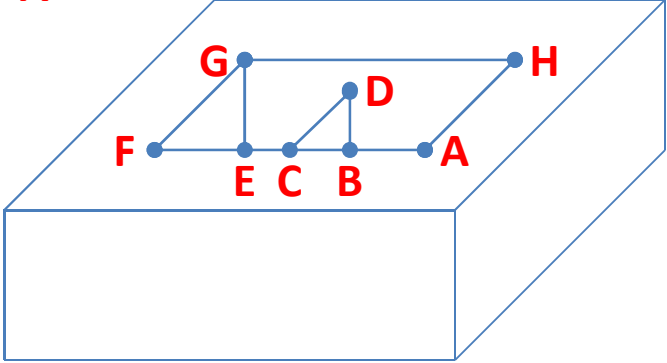
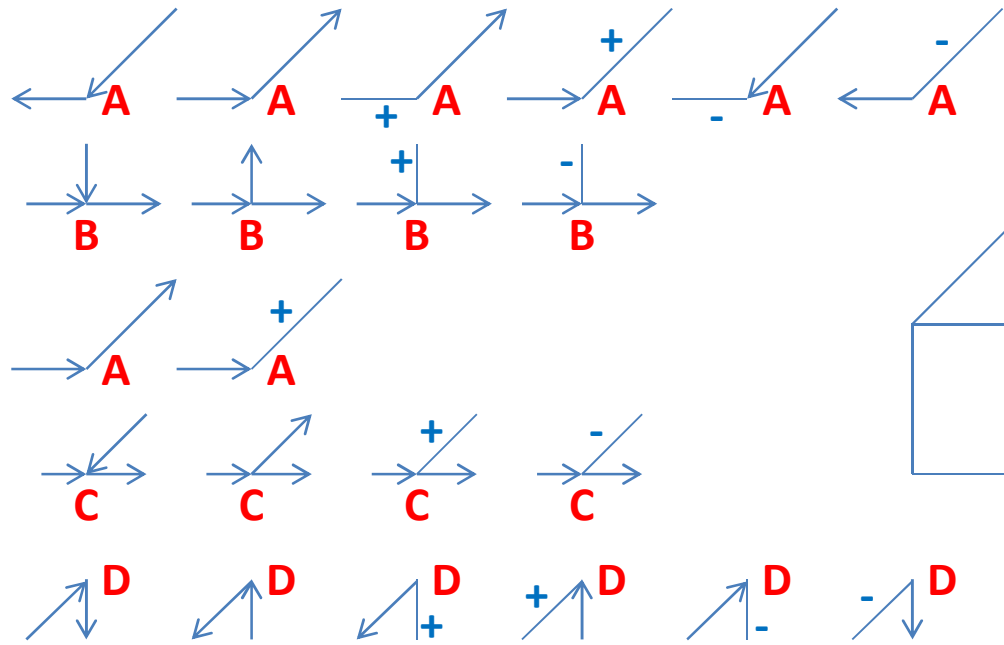
**A**

**B**

**A**

**C**

**D**



# Solution

**A**

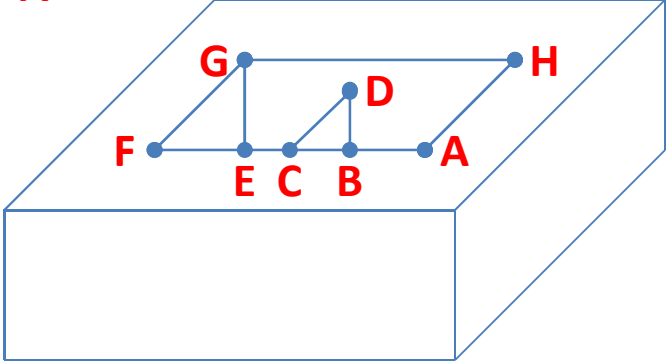
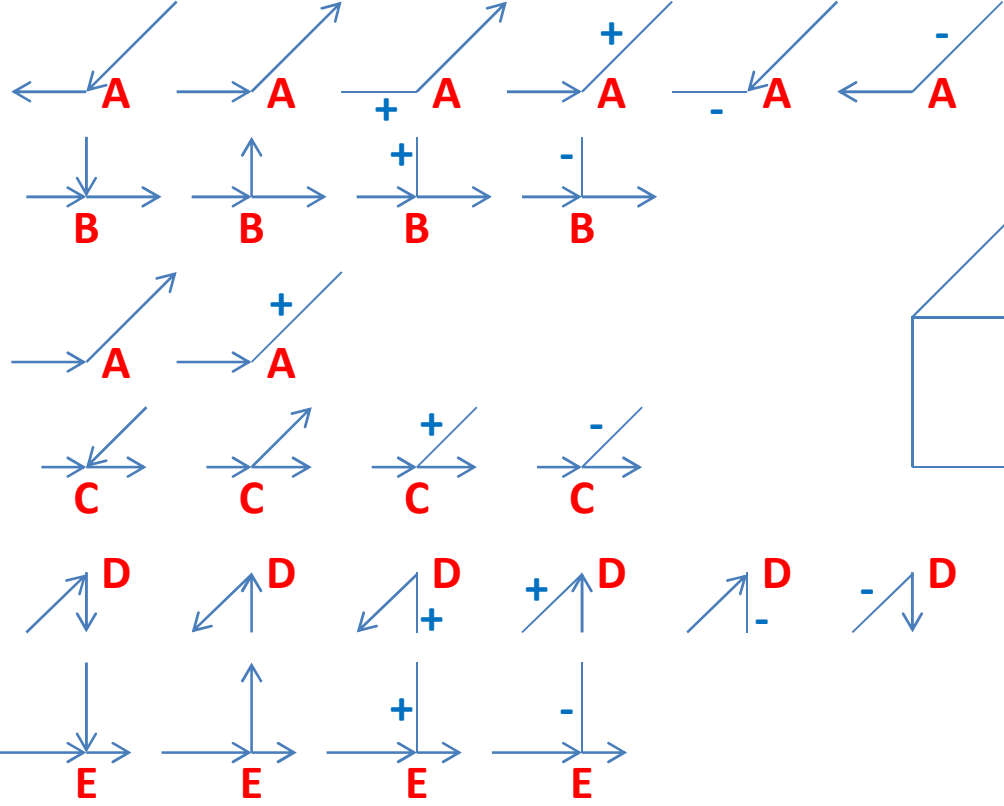
**B**

**A**

**C**

**D**

**E**



# Solution

**A**

**B**

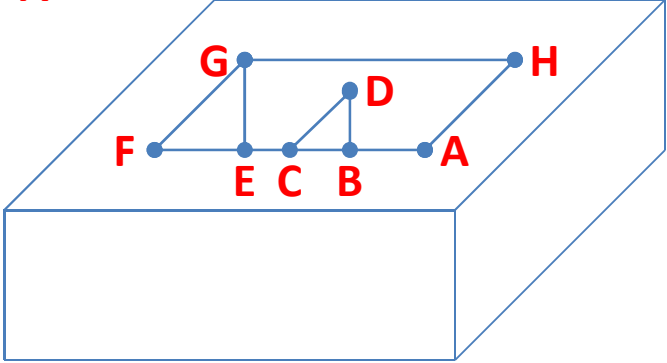
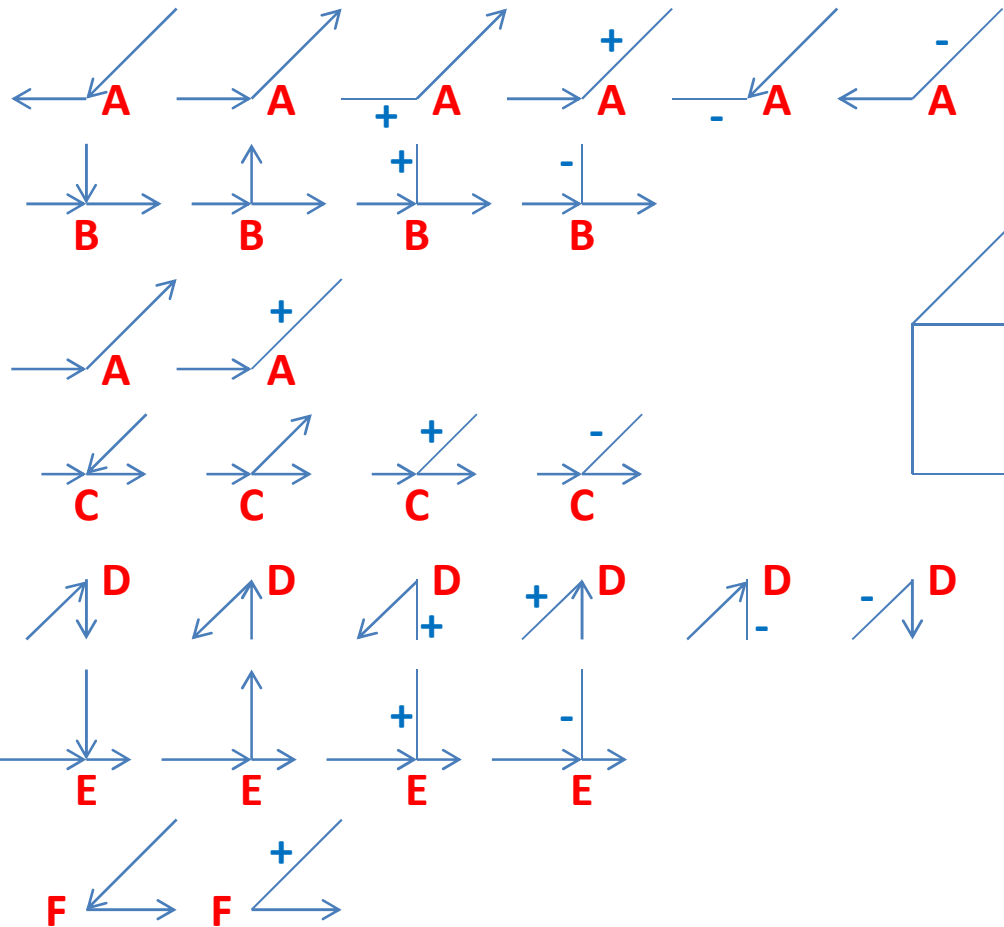
**A**

**C**

**D**

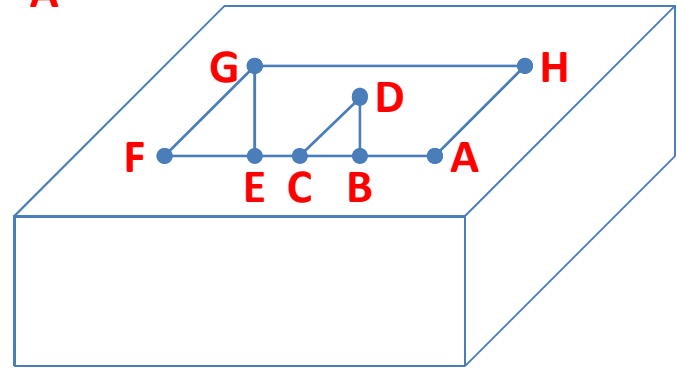
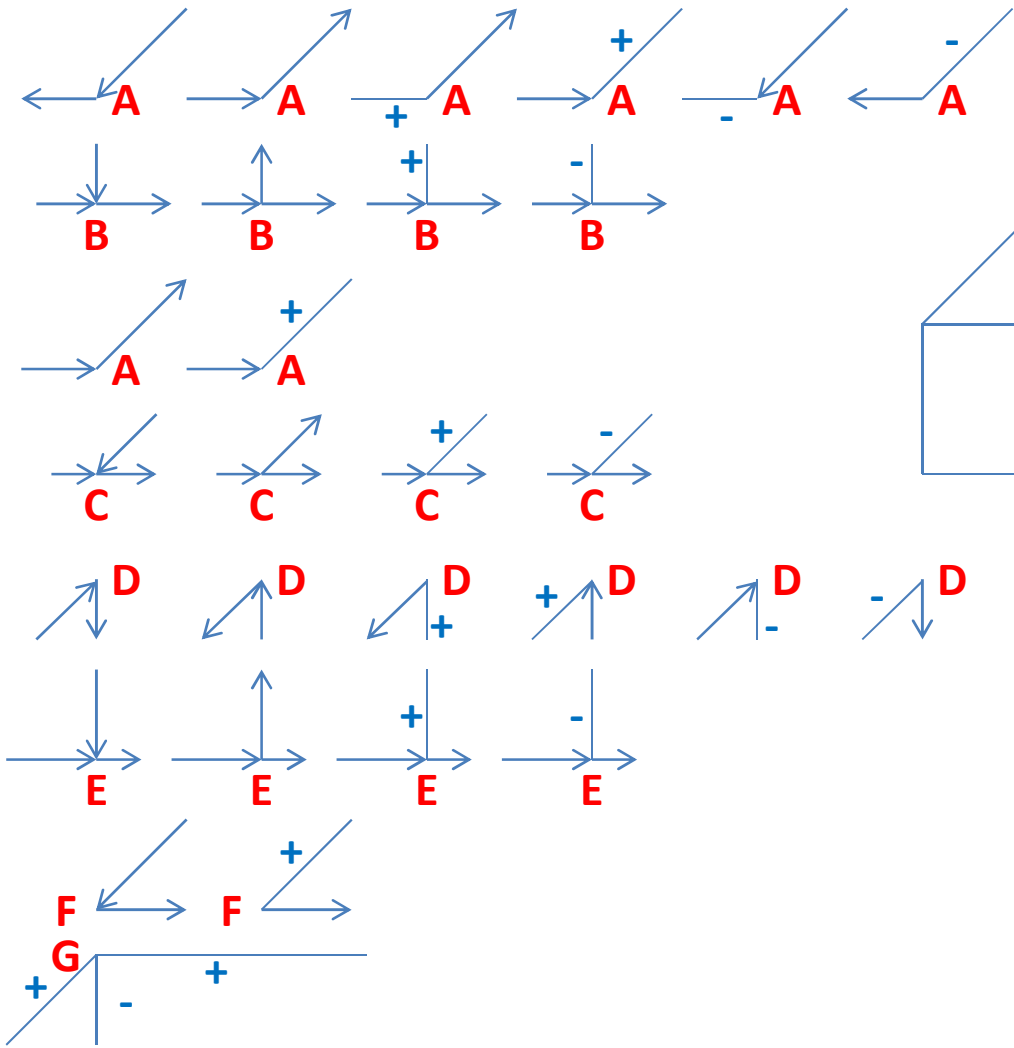
**E**

**F**



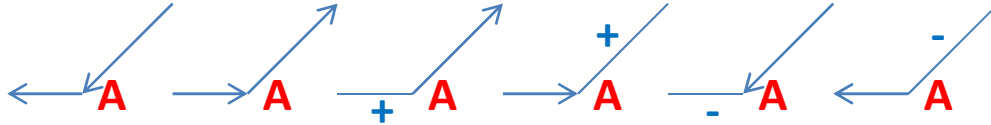
# Solution

- A
- B
- A
- C
- D
- E
- F
- G

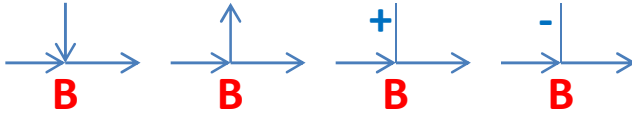


# Solution

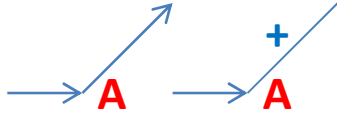
**A**



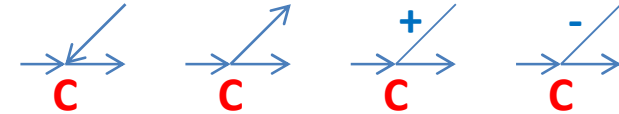
**B**



**A**



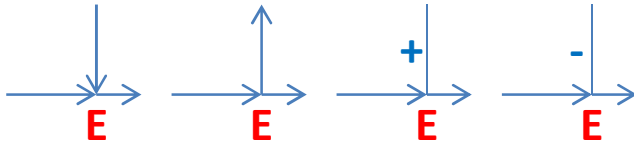
**C**



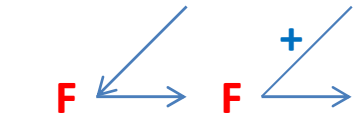
**D**



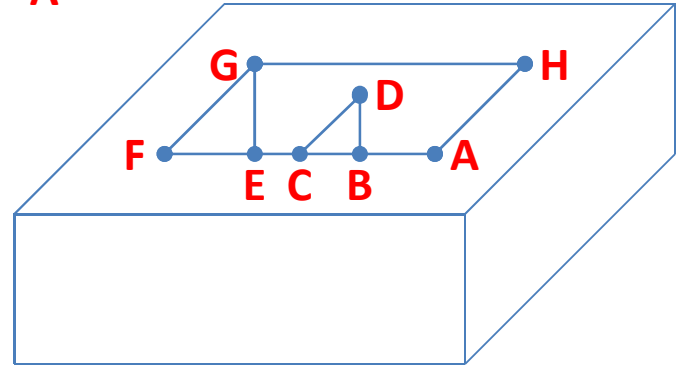
**E**



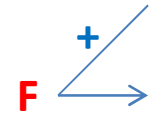
**F**



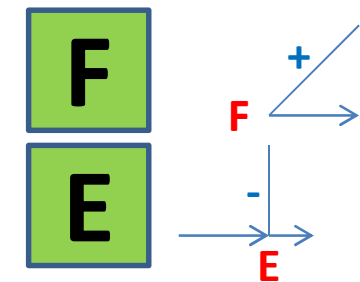
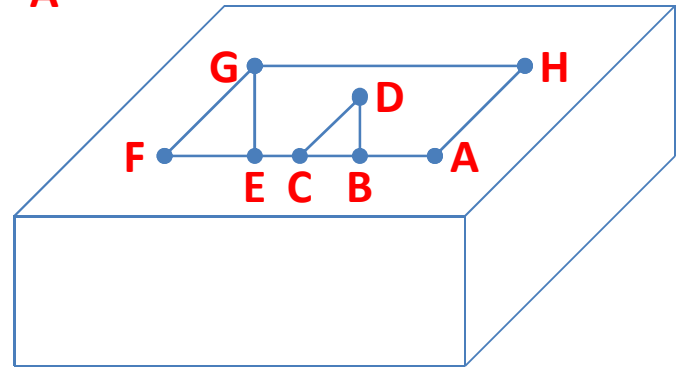
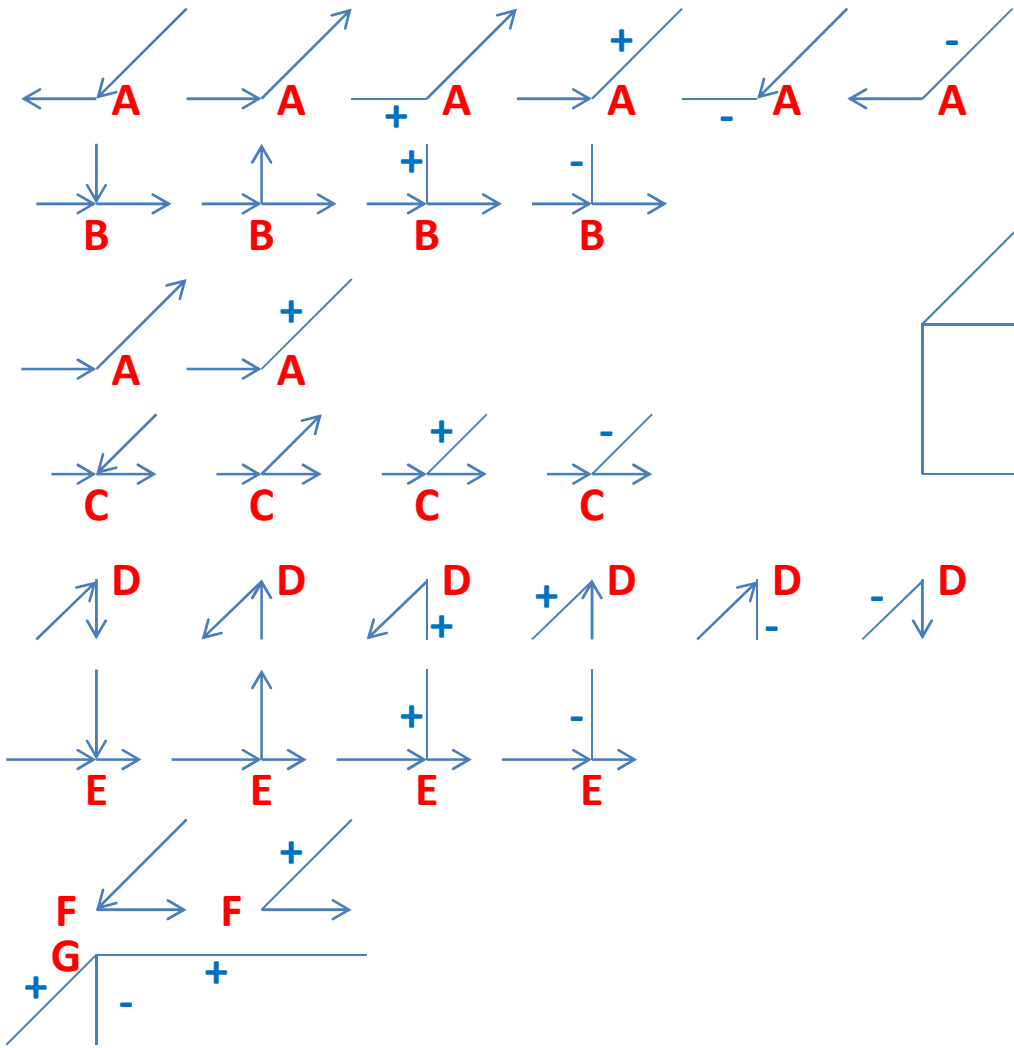
**G**



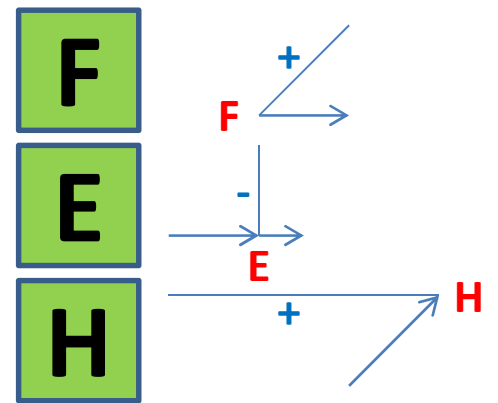
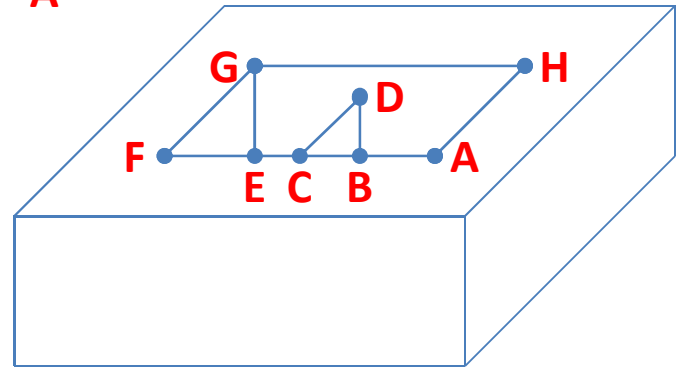
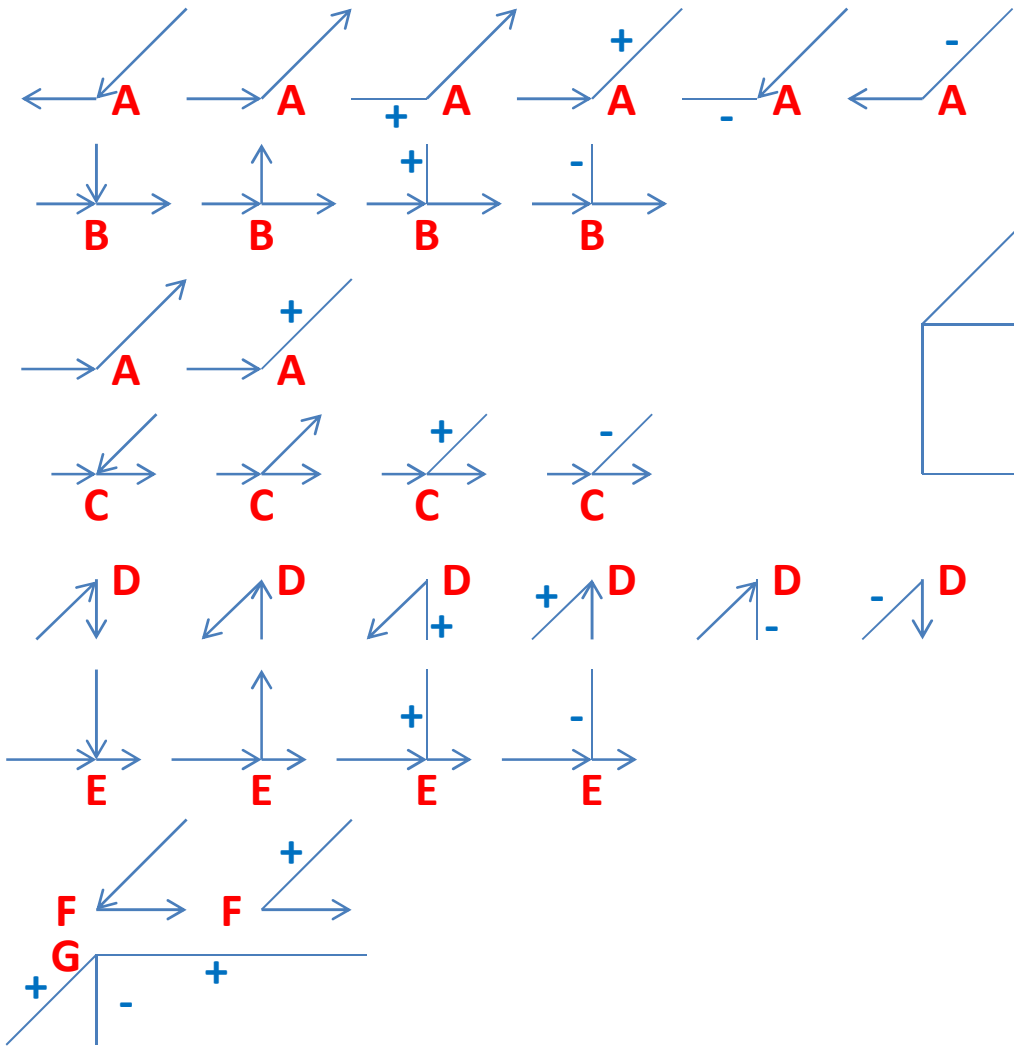
**F**



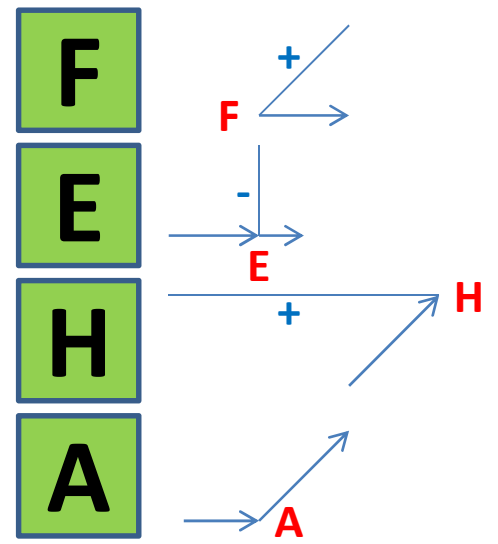
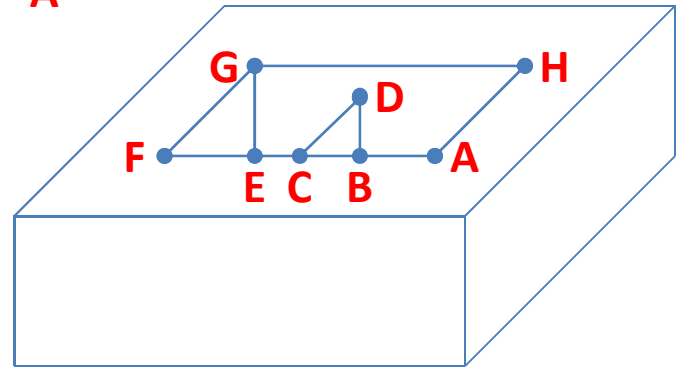
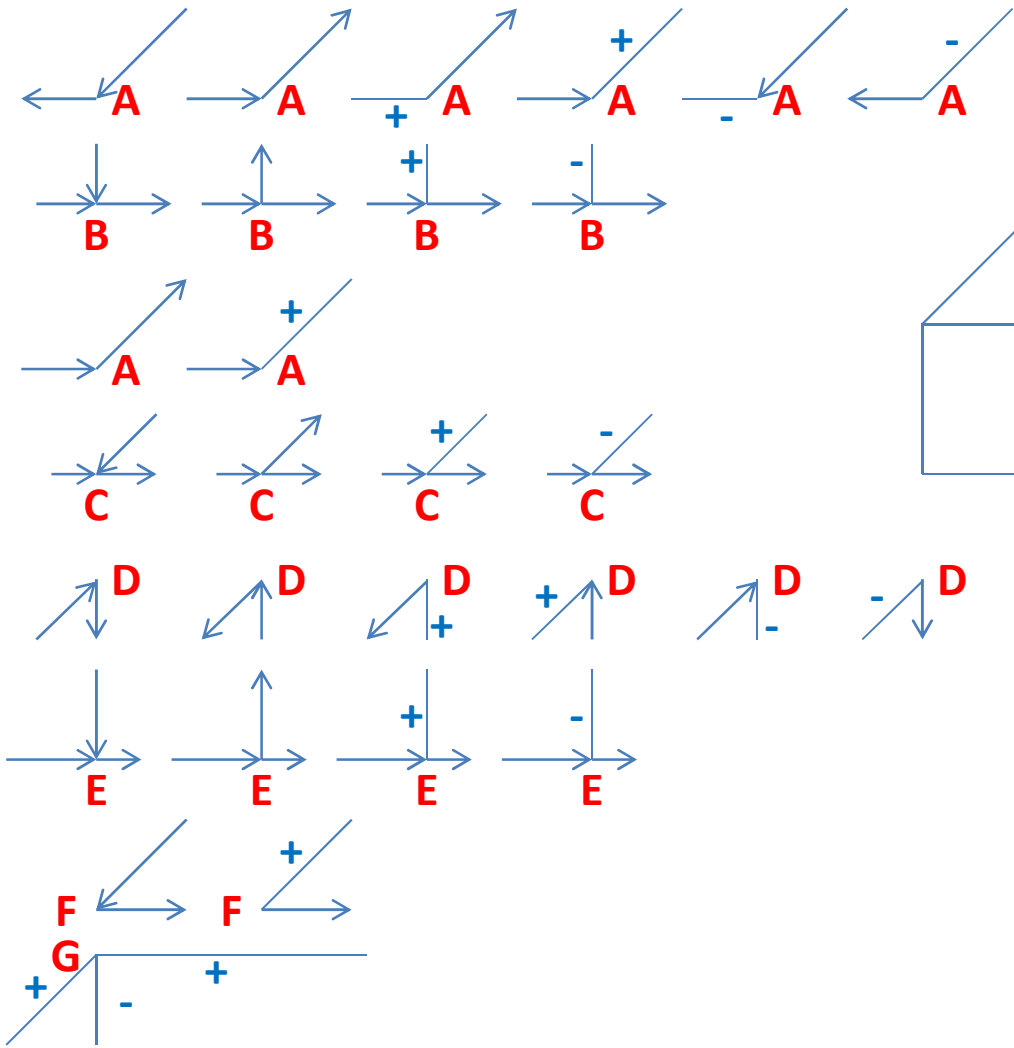
# Solution



# Solution

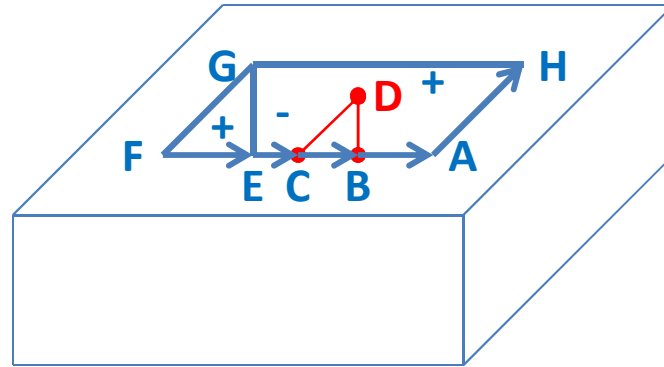


# Solution



# Solution

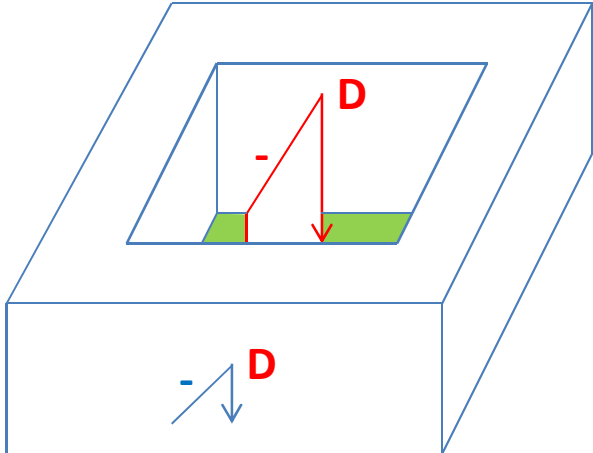
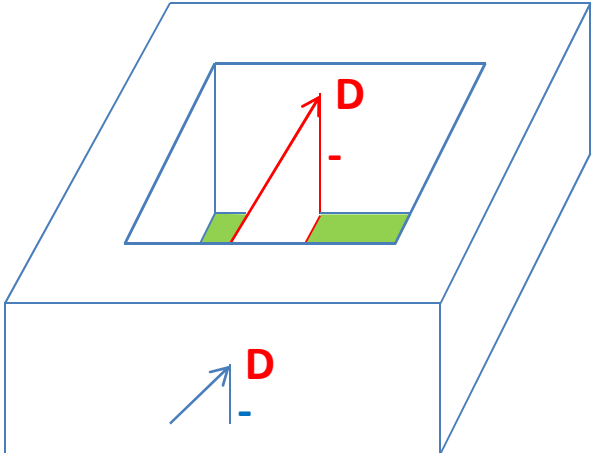
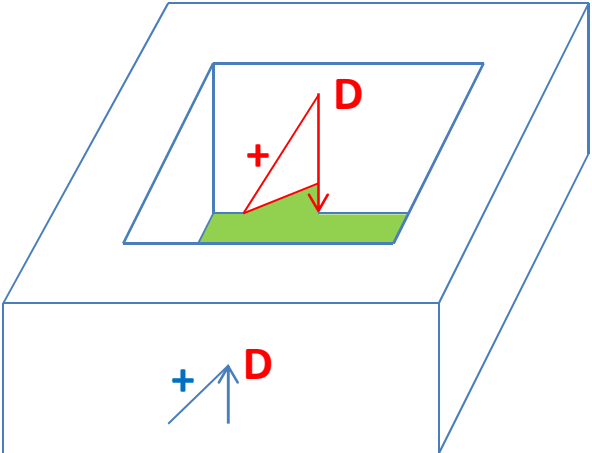
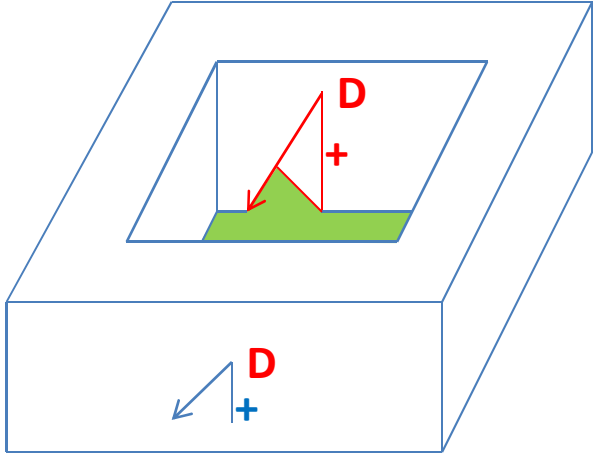
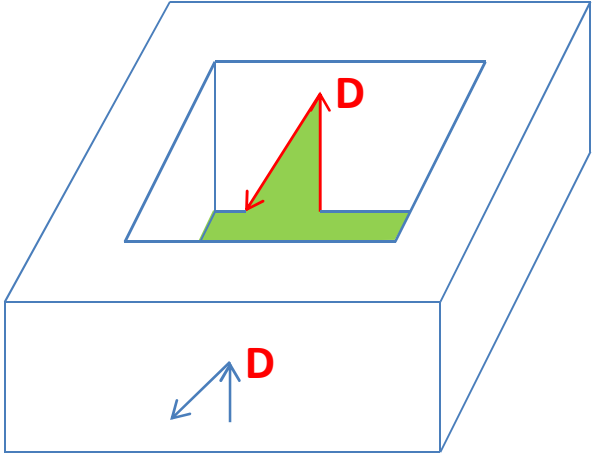
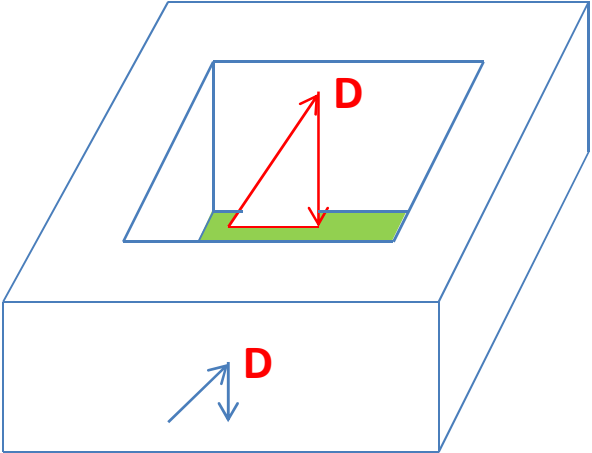
- We can determine **all nodes except for D**:



- **D** can still take **6 interpretations**:



# Solution



# Exercises: Artificial Intelligence

Constraint Processing II & Waltz:  
Waltz V

Constraint Processing II & Waltz: Waltz V

# **PROBLEM**

# Problem

- Prove the termination of the Waltz procedure

Constraint Processing II & Waltz: Waltz V

# **TERMINATION WALTZ**

# Termination Waltz

- ***Waltz's procedure terminates if***
  - No possibilities for some vertex      **OR**
  - No reduction of junction piles
- ***Waltz's procedure does not terminate if***
  - Only non-empty piles      **AND**
  - Reduction of piles possible
- **BUT**
  - Piles are finite  $\implies$  Number of iterations finite  
 $\implies$  **Waltz's procedure terminates**