

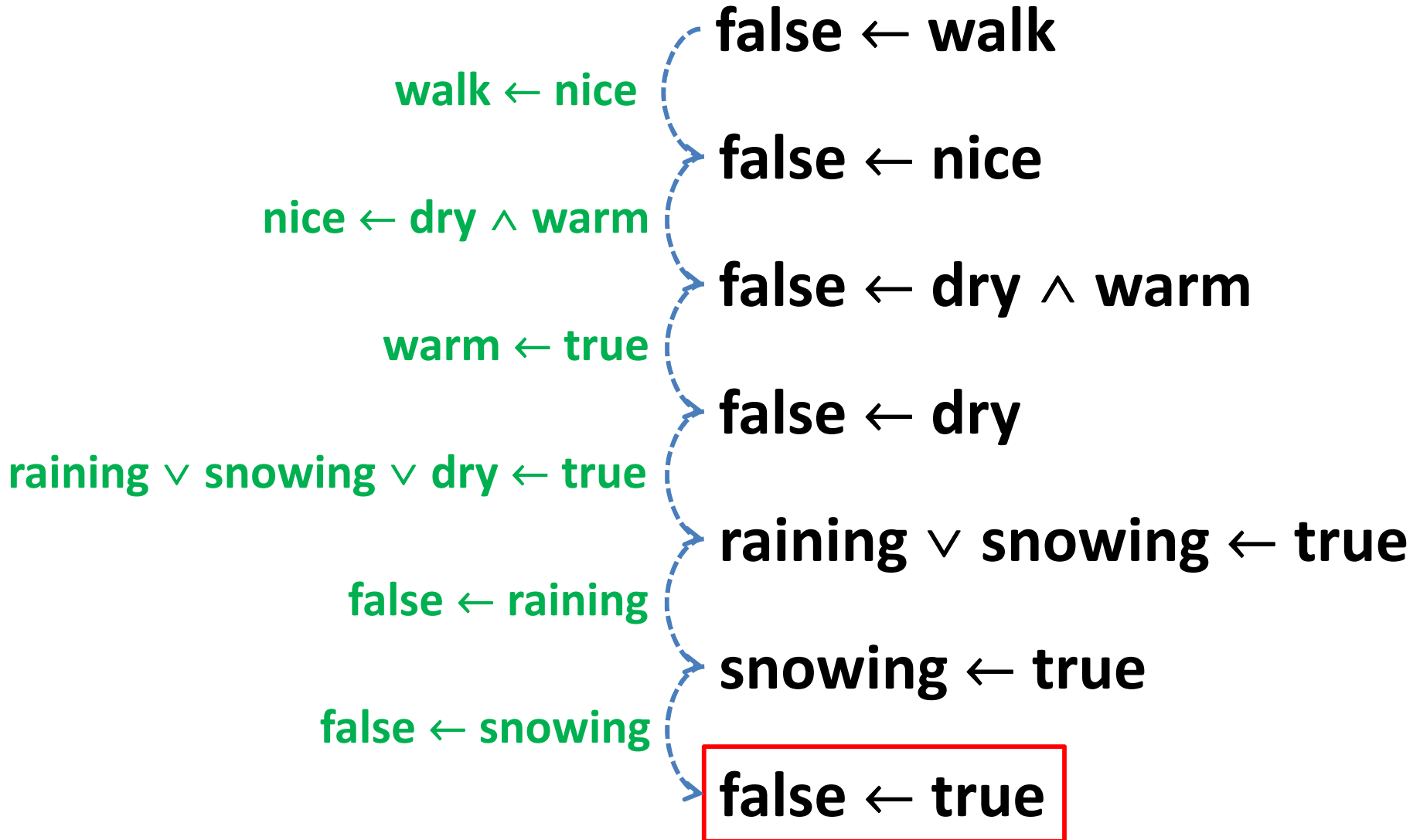
# Exercises: Artificial Intelligence

Automated Reasoning: Good to walk

# Solution

- *We assume that it is not good to walk:*
  - **false**  $\leftarrow$  walk
- *Given:*
  - raining  $\vee$  snowing  $\vee$  dry ( $\leftarrow$  true)
  - warm ( $\leftarrow$  true)
  - false  $\leftarrow$  raining
  - false  $\leftarrow$  snowing
  - walk  $\leftarrow$  nice
  - nice  $\leftarrow$  dry  $\wedge$  warm

# Solution



# Exercises: Artificial Intelligence

Automated Reasoning: MGU

# Solution

MGU:  $\{x/f(A), w/f(A), y/A\}$

Result:  $p(f(A), f(A), g(z, A))$

- *What is the m.g.u. of:  $p(f(y), w, g(z, y)) = p(x, x, g(z, A))$* 
  - *Init:  $p(f(y), w, g(z, y)) = p(x, x, g(z, A))$*
  - *Case 5:  $f(y) = x, w = x, g(z, y) = g(z, A)$*
  - *Case 1:  $x = f(y), w = x, g(z, y) = g(z, A)$*
  - *Case 4:  $x = f(y), w = f(y), g(z, y) = g(z, A)$*
  - *Case 5:  $x = f(y), w = f(y), z = z, y = A$*
  - *Case 2:  $x = f(y), w = f(y), y = A$*
  - **Case 4:  $x = f(A), w = f(A), y = A$**

# Solution

- *What is the m.g.u. of:  $p(A, x, f(g(y))) = p(z, f(z), f(A))$* 
  - *Init:  $p(A, x, f(g(y))) = p(z, f(z), f(A))$*
  - *Case 5:  $A = z, x = f(z), f(g(y)) = f(A)$*
  - *Case 1:  $z = A, x = f(z), f(g(y)) = f(A)$*
  - *Case 4:  $z = A, x = f(A), f(g(y)) = f(A)$*
  - *Case 5:  $z = A, x = f(A), g(y) = A$*
  - ***Case 5: stop := true***

# Solution

- *What is the m.g.u. of:  $q(x,x) = q(y,f(y))$* 
  - *Init:  $q(x,x) = q(y,f(y))$*
  - *Case 5:  $x = y, x = f(y)$*
  - *Case 4:  $x = y, y = f(y)$*
  - ***Case 3: stop := true***

# Solution

MGU:  $\{x/g(f(a),f(a)), u/f(a), v/f(a)\}$

Result:  $f(g(f(a),f(a)),g(f(a), f(a)))$

- *What is the m.g.u. of:  $f(x,g(f(a),u)) = f(g(u,v),x)$* 
  - *Init:  $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Case 5:  $x = g(u,v), g(f(a),u) = x$*
  - *Case 4:  $x = g(u,v), g(f(a),u) = g(u,v)$*
  - *Case 5:  $x = g(u,v), f(a) = u, u = v$*
  - *Case 1:  $x = g(u,v), u = f(a), u = v$*
  - *Case 4:  $x = g(f(a),v), u = f(a), f(a) = v$*
  - *Case 1:  $x = g(f(a),v), u = f(a), v = f(a)$*
  - **Case 4:  $x = g(f(a),f(a)), u = f(a), v = f(a)$**

# Exercises: Artificial Intelligence

Automated Reasoning: Resolution

# Solution

- *Assumption: Peter has no mother-in-law*
  - $\text{false} \leftarrow \text{mother-in-law}(x, \text{Peter})$
- *Given:*
  - $\text{mother-in-law}(x, y) \leftarrow \text{mother}(x, z) \wedge \text{married}(z, y)$
  - $\text{mother}(x, y) \leftarrow \text{female}(x) \wedge \text{parent}(x, y)$
  - $\text{female}(\text{An}) \leftarrow \text{true}$
  - $\text{parent}(\text{An}, \text{Maria}) \leftarrow \text{true}$
  - $\text{married}(\text{Maria}, \text{Peter}) \leftarrow \text{true}$

# Solution

- $\text{false} \leftarrow \text{mother-in-law}(x, \text{Peter})$ 
  - $\text{mother-in-law}(x', y') \leftarrow \text{mother}(x', z') \wedge \text{married}(z', y')$
  - $\{x'/x, y'/\text{Peter}\}$
- $\text{false} \leftarrow \text{mother}(x, z') \wedge \text{married}(z', \text{Peter})$

# Solution

- $\text{false} \leftarrow \text{mother-in-law}(x, \text{Peter})$
- $\text{false} \leftarrow \text{mother}(x, z') \wedge \text{married}(z', \text{Peter})$ 
  - $\text{mother}(x', y') \leftarrow \text{female}(x') \wedge \text{parent}(x', y')$
  - $\{x'/x, y'/z'\}$
- $\text{false} \leftarrow \text{female}(x) \wedge \text{parent}(x, z') \wedge \text{married}(z', \text{Peter})$

# Solution

- $\text{false} \leftarrow \text{mother-in-law}(x, \text{Peter})$
- $\text{false} \leftarrow \text{mother}(x, z') \wedge \text{married}(z', \text{Peter})$
- $\text{false} \leftarrow \text{female}(x) \wedge \text{parent}(x, z') \wedge \text{married}(z', \text{Peter})$ 
  - $\text{female}(An)$
  - $\{x/An\}$
- $\text{false} \leftarrow \text{parent}(An, z') \wedge \text{married}(z', \text{Peter})$

# Solution

- $\text{false} \leftarrow \text{mother-in-law}(x, \text{Peter})$
- $\text{false} \leftarrow \text{mother}(x, z') \wedge \text{married}(z', \text{Peter})$
- $\text{false} \leftarrow \text{female}(x) \wedge \text{parent}(x, z') \wedge \text{married}(z', \text{Peter})$
- $\text{false} \leftarrow \text{parent}(\text{An}, z') \wedge \text{married}(z', \text{Peter})$ 
  - $\text{parent}(\text{An}, \text{Maria})$
  - $\{z' / \text{Maria}\}$
- $\text{false} \leftarrow \text{married}(\text{Maria}, \text{Peter})$

# Solution

$\{x/An\}$

- false  $\leftarrow$  mother-in-law(x,Peter)
- false  $\leftarrow$  mother(x,z')  $\wedge$  married(z',Peter)
- false  $\leftarrow$  female(x)  $\wedge$  parent(x,z')  $\wedge$  married(z',Peter)
- false  $\leftarrow$  parent(An,z')  $\wedge$  married(z',Peter)
- false  $\leftarrow$  married(Maria,Peter)
  - married(Maria,Peter)
- false  $\leftarrow$  true ( $\square$ )

# Solution

- *Assumption: “There is no valid colouring”*
  - $false \leftarrow nb(b,g), nb(g,n), nb(n,b)$
- *Given:*
  - $c(R) \leftarrow true$
  - $c(G) \leftarrow true$
  - $c(B) \leftarrow true$
  - $nb(x,y) \leftarrow c(x), c(y), diff(x,y)$ 
    - $diff/2$  succeeds when arguments cannot be unified

# Solution

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$ 
  - $\text{nb}(x',y') \leftarrow c(x'), c(y'), \text{diff}(x',y')$
  - $\{x'/b, y'/g\}$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$

# Solution

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$ 
  - $\text{nb}(x',y') \leftarrow c(x'), c(y'), \text{diff}(x',y')$
  - $\{x'/g, y'/n\}$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$

# Solution

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$ 
  - $\text{nb}(x',y') \leftarrow c(x'), c(y'), \text{diff}(x',y')$
  - $\{x'/n, y'/b\}$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,b)$

# Solution

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,b)$ 
  - $c(R)$
  - $\{b/R\}$
- $\text{false} \leftarrow c(g) \wedge \text{diff}(R,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,R)$

# Solution

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,b)$
- $\text{false} \leftarrow c(g) \wedge \text{diff}(R,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,R)$ 
  - $c(G)$
  - $\{g/G\}$
- $\text{false} \leftarrow \text{diff}(R,G) \wedge c(n) \wedge \text{diff}(G,n) \wedge \text{diff}(n,R)$

# Solution

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,b)$
- $\text{false} \leftarrow c(g) \wedge \text{diff}(R,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,R)$
- $\text{false} \leftarrow \text{diff}(R,G) \wedge c(n) \wedge \text{diff}(G,n) \wedge \text{diff}(n,R)$ 
  - $c(B)$
  - $\{n/B\}$
- $\text{false} \leftarrow \text{diff}(R,G) \wedge \text{diff}(G,B) \wedge \text{diff}(B,R)$

# Solution

$\{b/R, g/G, n/B\}$

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,b)$
- $\text{false} \leftarrow c(g) \wedge \text{diff}(R,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,R)$
- $\text{false} \leftarrow \text{diff}(R,G) \wedge c(n) \wedge \text{diff}(G,n) \wedge \text{diff}(n,R)$
- $\text{false} \leftarrow \text{diff}(R,G) \wedge \text{diff}(G,B) \wedge \text{diff}(B,R)$ 
  - Built-in  $\text{diff}/2$ : succeeds for different arguments
- $\text{false} \leftarrow \text{true} (\square)$

# Alternative solution

**{b/B,g/G,n/R}**

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,b)$
- $\text{false} \leftarrow c(g) \wedge \text{diff}(\underline{\mathbf{B}},g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,\underline{\mathbf{B}})$
- $\text{false} \leftarrow \text{diff}(\underline{\mathbf{B}},G) \wedge c(n) \wedge \text{diff}(G,n) \wedge \text{diff}(n,\underline{\mathbf{B}})$
- $\text{false} \leftarrow \text{diff}(\underline{\mathbf{B}},G) \wedge \text{diff}(G,\underline{\mathbf{R}}) \wedge \text{diff}(\underline{\mathbf{R}},\underline{\mathbf{B}})$ 
  - Built-in `diff/2`: succeeds for different arguments
- $\text{false} \leftarrow \text{true} (\square)$

# Or consistency = Continue search

- $\text{false} \leftarrow \text{nb}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge \text{nb}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{nb}(n,b)$
- $\text{false} \leftarrow c(b) \wedge c(g) \wedge \text{diff}(b,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,b)$
- $\text{false} \leftarrow c(g) \wedge \text{diff}(R,g) \wedge c(n) \wedge \text{diff}(g,n) \wedge \text{diff}(n,R)$
- $\text{false} \leftarrow \text{diff}(R,\underline{R}) \wedge c(n) \wedge \text{diff}(\underline{R},n) \wedge \text{diff}(n,R)$
- $\text{false} \leftarrow \text{diff}(R,\underline{R}) \wedge \text{diff}(\underline{R},B) \wedge \text{diff}(B,R)$ 
  - $\text{diff}(R,R)$  is false
- $\text{false} \leftarrow \text{false}$

# Exercises: Artificial Intelligence

Automated Reasoning: Predicate  
Resolution

# Solution

- Formula in implicative normal form:

$$- \forall x p(x) \vee \neg r(f(x))$$

- $p(x) \leftarrow r(f(x))$

$$- \forall x \forall y r(f(x)) \vee r(f(f(y)))$$

- $r(f(x)) \vee r(f(f(y))) (\leftarrow \text{true})$

- Assumption

$$\neg [\forall x \exists y p(f(x)) \wedge r(y)] \Leftrightarrow \exists x \forall y \neg [p(f(x)) \wedge r(y)] \Leftrightarrow$$

$$\forall y \neg [p(f(A)) \wedge r(y)] \Leftrightarrow \text{false} \leftarrow p(f(A)) \wedge r(y)$$

# Solution

- $\text{false} \leftarrow p(f(A)) \wedge r(y)$ 
  - $p(x') \leftarrow r(f(x'))$
  - $\{x'/f(A)\}$
- $\text{false} \leftarrow r(f(f(A))) \wedge r(y)$

# Solution

- $\text{false} \leftarrow p(f(A)) \wedge r(y)$
- $\text{false} \leftarrow r(f(f(A))) \wedge r(y)$ 
  - Factoring:  $\text{mgu}(r(f(f(A))) = r(y)) = \{y/f(f(A))\}$
- $\text{false} \leftarrow r(f(f(A))) \wedge r(f(f(A)))$

# Solution

$$\{y/f(f(A))\}$$

- $\text{false} \leftarrow p(f(A)) \wedge r(y)$
- $\text{false} \leftarrow r(f(f(A))) \wedge r(y)$
- $\text{false} \leftarrow r(f(f(A))) \wedge r(f(f(A)))$ 
  - $r(f(x')) \vee r(f(f(y')))$  ( $\leftarrow \text{true}$ )
    - Factoring:  $\text{mgu}(r(f(x')) = r(f(f(y')))) = \{x'/f(y')\}$
  - $r(f(f(y')))$  ( $\leftarrow \text{true}$ )
  - $\{y'/A\}$
- $\text{false} \leftarrow \text{true} (\square)$